A.P. Calculus Practice Exam - Chain Rule,Trig, Implicit

Find the derivative of each function. Be sure to indicate the derivative in proper notation. Do only the most obvious simplifications.

1. $y = \frac{3}{7}\cos x$	2. $y = \csc 5x$	3. $y = -3\sin^2 x$
4. $y = \tan 7x^2$	5. $y = 2x\cot x$	6. $y = \frac{x}{2\sin x}$
7. $y = \tan 8x + \cos\frac{1}{8}x$	8. $y = \cos^3 x^3$	9. $y = \cos(\sin x)$
10. $y = \cos^2(3x)\sin(4x)$	11. $y = \sin\sqrt[3]{3x}$	12. $y = \sqrt[3]{\sin 3x}$

Find the derivative. Simplify according to the rules established in class.

13. $f(x)=\left(x^3-4x^2+1\right)^6$

14. $f(x)=5x\left(x^2+1\right)^3$

15. $y=\dfrac{1}{\sqrt{x^2-7x+3}}$

16. $f(x)=\left(\dfrac{x+1}{3x+1}\right)^3$

Find the slope of the tangent line to the following graphs at the given points by using implicit differentiation.

17. $x^2+2y^2=17$ at (3,2)

18. $(x-y)^2+3x=-6y$ at (1, -2)

19. Find $\dfrac{dy}{dx}$ implicitly

$x^3-3xy-2y=4$

20. Find the point(s) where the following graph has a **horizontal** tangent line.

$4x^2+y^2-8x+4y+4=0$

21. Find the equation of the tangent line to $y=\dfrac{\cos x-\tan x}{\cos x}$ at $x=0$

A.P. Calculus Practice Exam - Chain Rule,Trig, Implicit - Solutions

Find the derivative of each function. Be sure to indicate the derivative in proper notation. Do only the most obvious simplifications.

1. $y=\frac{3}{7}\cos x$

Ans: $y'=-\frac{3}{7}\sin x$

2. $y=\csc 5x$

Ans: $y'=-5\csc(5x)\cot(5x)$

3. $y=-3\sin^2 x$

Ans: $y'=-6\sin x\cos x$

4. $y=\tan 7x^2$

Ans: $y'=14x\sec^2\left(7x^2\right)$

5. $y=2x\cot x$

Ans: $y'=-2x\csc^2 x+2\cot x$

6. $y=\frac{x}{2\sin x}$

Ans: $y'=\frac{\sin x-x\cos x}{2\sin^2 x}$

7. $y=\tan 8x+\cos\frac{1}{8}x$

Ans: $y'=8\sec^2(8x)-\frac{1}{8}\sin\left(\frac{1}{8}x\right)$

8. $y=\cos^3 x^3$

Ans: $y'=-9x^2\left[\cos\left(x^3\right)\right]^2\left[\sin\left(x^3\right)\right]$

9. $y=\cos(\sin x)$

$y'=-\sin(\sin x)(\cos x)$

10. $y=\cos^2(3x)\sin(4x)$

Ans:

$y'=\cos^2(3x)\cos(4x)\cdot 4+$
$\sin(4x)(2)\cos(3x)(-\sin 3x)\cdot 3$

$y'=4\cos^2(3x)\cos(4x)+$
$-6\sin(4x)\cos(3x)(\sin 3x)$

11. $y=\sin\sqrt[3]{3x}$

Ans:

$y'=\cos(3x)^{1/3}\left(\frac{1}{3}(3x)^{-2/3}(3)\right)$

$y'=\frac{\cos(3x)^{1/3}}{(3x)^{2/3}}$

12. $y=\sqrt[3]{\sin 3x}$

$y=\left[\sin(3x)\right]^{1/3}$

Ans: $y'=\frac{1}{3}\left[\sin(3x)\right]^{-2/3}\cos(3x)\cdot 3$

$y'=\frac{\cos(3x)}{\left[\sin(3x)\right]^{2/3}}$

Find the derivative. Simplify according to the rules established in class.

13. $f(x)=\left(x^3-4x^2+1\right)^6$

$$\boxed{f'(x)=6\left(x^3-4x^2+1\right)^5\left(3x^2-8x\right)}$$

14. $f(x)=5x\left(x^2+1\right)^3$

$$\boxed{f'(x)=30x^2\left(x^2+1\right)^2+5\left(x^2+1\right)^3}$$

15. $y=\dfrac{1}{\sqrt{x^2-7x+3}}$

$$\boxed{y'=\frac{7-2x}{2\left(x^2-7x+3\right)^{3/2}}}$$

16. $f(x)=\left(\dfrac{x+1}{3x+1}\right)^3$

$$\boxed{f'(x)=3\left(\frac{x+1}{3x+1}\right)^2\left[\frac{-2}{(3x+1)^2}\right]}$$

Find the slope of the tangent line to the following graphs at the given points by using implicit differentiation.

17. $x^2+2y^2=17$ at (3,2)

:
$$\boxed{\begin{aligned}&2x+4y\frac{dy}{dx}=0\\&\frac{dy}{dx}=\frac{-x}{2y}=\frac{-3}{4}\end{aligned}}$$

18. $(x-y)^2+3x=-6y$ at (1, -2)

$$\boxed{\begin{aligned}&2(x-y)\left(1-\frac{dy}{dx}\right)+3=-6\frac{dy}{dx}\\&2(3)\left(1-\frac{dy}{dx}\right)+3=-6\frac{dy}{dx}\\&\frac{dy}{dx}=DNE\end{aligned}}$$

19. Find $\dfrac{dy}{dx}$ implicitly

$x^3-3xy-2y=4$

$$\boxed{\begin{aligned}&3x^2-3\left(x\frac{dy}{dx}+y\right)-2\frac{dy}{dx}=0\\&3x^2-3y=\frac{dy}{dx}(3x+2)\\&\frac{dy}{dx}=\frac{3x^2-3y}{3x+2}\end{aligned}}$$

20. Find the point(s) where the following graph has a **horizontal** tangent line.

$4x^2+y^2-8x+4y+4=0$

$$\boxed{\begin{aligned}&8x+2y\frac{dy}{dx}-8+4\frac{dy}{dx}=0\\&\frac{dy}{dx}=\frac{8x-8}{-4-2y}=0\Rightarrow x=1\\&4+y^2-8+4y+4=0\Rightarrow y=0,y=-4\\&(1,0),(1,-4)\end{aligned}}$$

21. Find the equation of the tangent line to $y=\dfrac{\cos x-\tan x}{\cos x}$ at $x=0$

$$\boxed{\begin{aligned}&y=\frac{\cos x-\tan x}{\cos x}\\&y'=\frac{\cos x\left(-\sin x-\sec^2 x\right)-(\cos x-\tan x)(-\sin x)}{\cos^2 x}\\&y'(0)=\frac{1(0-1)-(1-0)(0)}{1}=-1\\&y-1=-1(x-0)\Rightarrow y=1-x\end{aligned}}$$