(1) $f(x) = x^3 - x + 5$

continuous on $[0,1]$? ✓

diff. on $(0,1)$? ✓

$$f'(x) = 3x^2 - 1 \stackrel{?}{=} 0$$

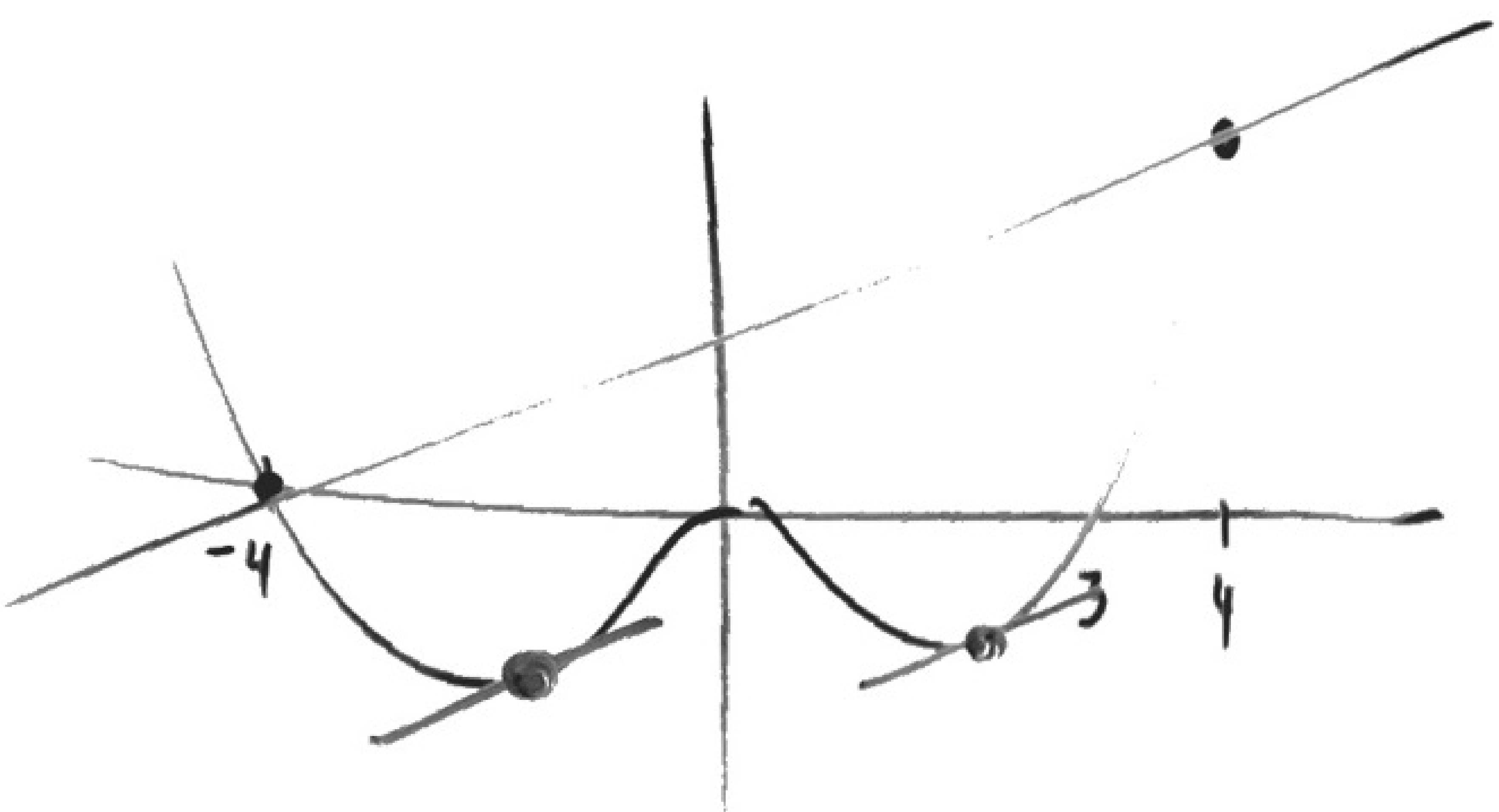
$$3x^2 = 1$$

$$x^2 = 1/3$$

$$x = \pm\sqrt{1/3}$$

$\boxed{x = \sqrt{1/3} \text{ satisfies R.T. on } [0,1].}$

(2)



③ $f(x) = 8x - x^2 + 1$ on $[-1, 3]$

C on $[-1,3]$? ✓

D on $(-1,3)$? ✓

$f'(x) = 8 - 2x \overset{?}{=} 6$ $\quad m_{sec} = \dfrac{f(3) - f(-1)}{3 - -1} = \dfrac{(24 - 9 + 1) - (-8 - 1 + 1)}{4}$

$= 6$

$-2x = -2$

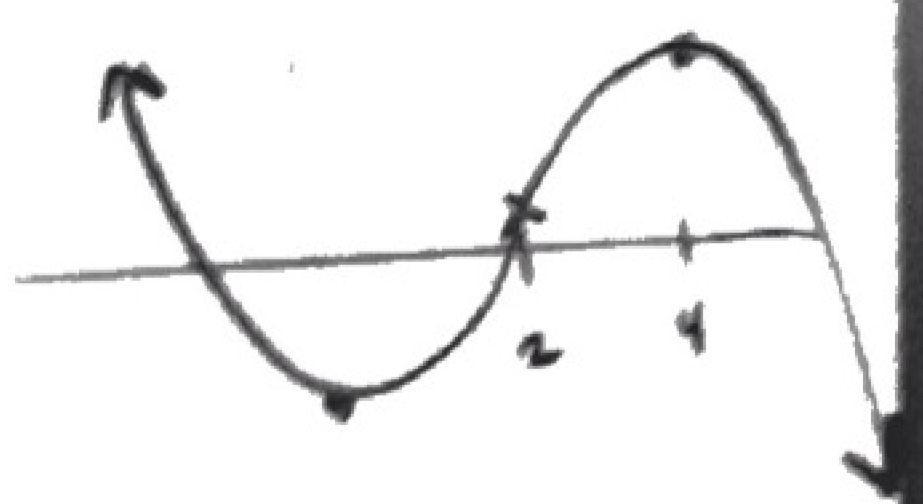
$\boxed{x = 1}$ MVT is satisfied at $x = 1$ on the interval $[-1, 3]$.

④

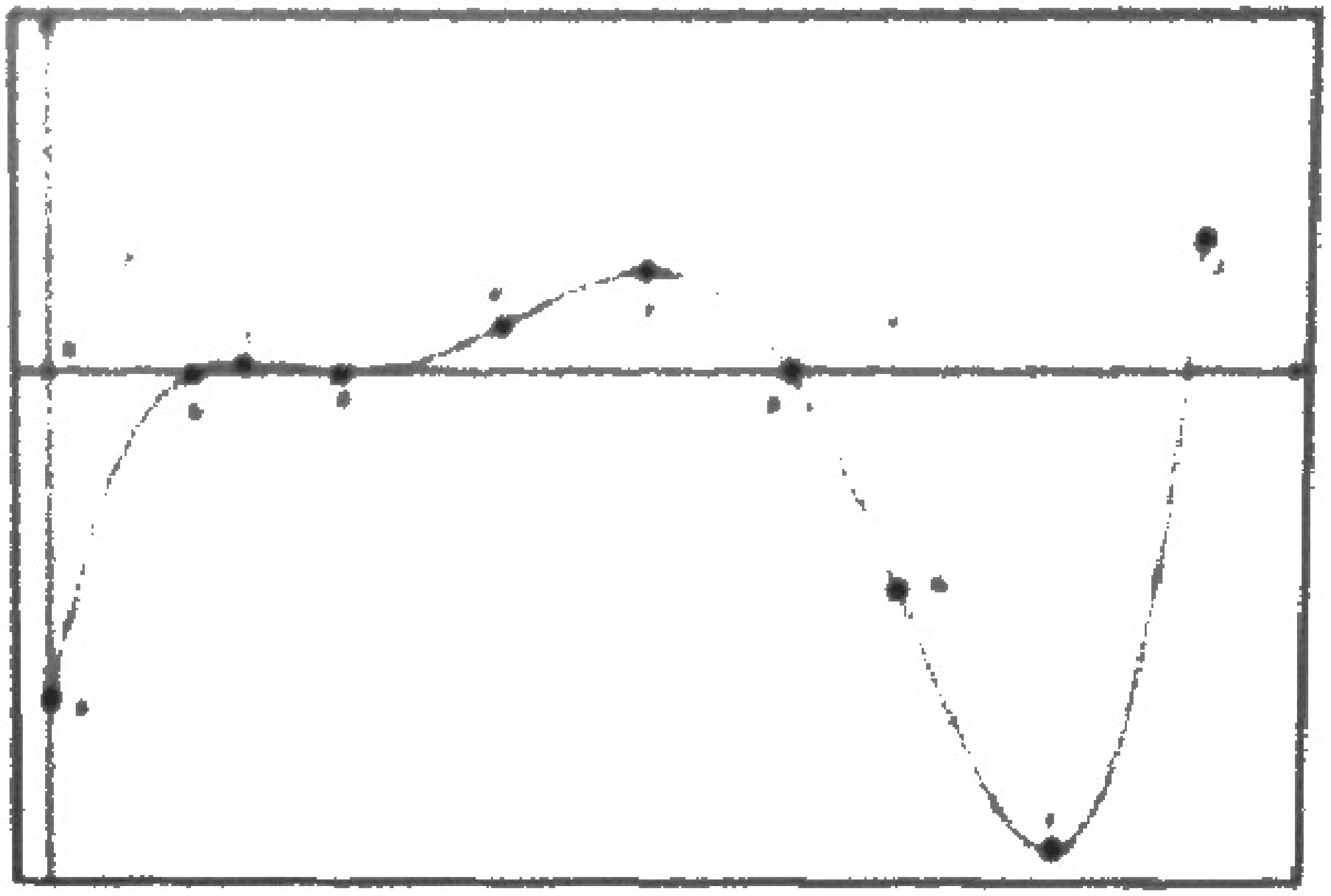
f': dec $-$ rel. min 0 at $x = 0$, inc $+$, rel. max 0 at $x = 4$, dec $-$

f'': CU $+$, I.P. 0 at $x = 2$, CD $-$

f(x)

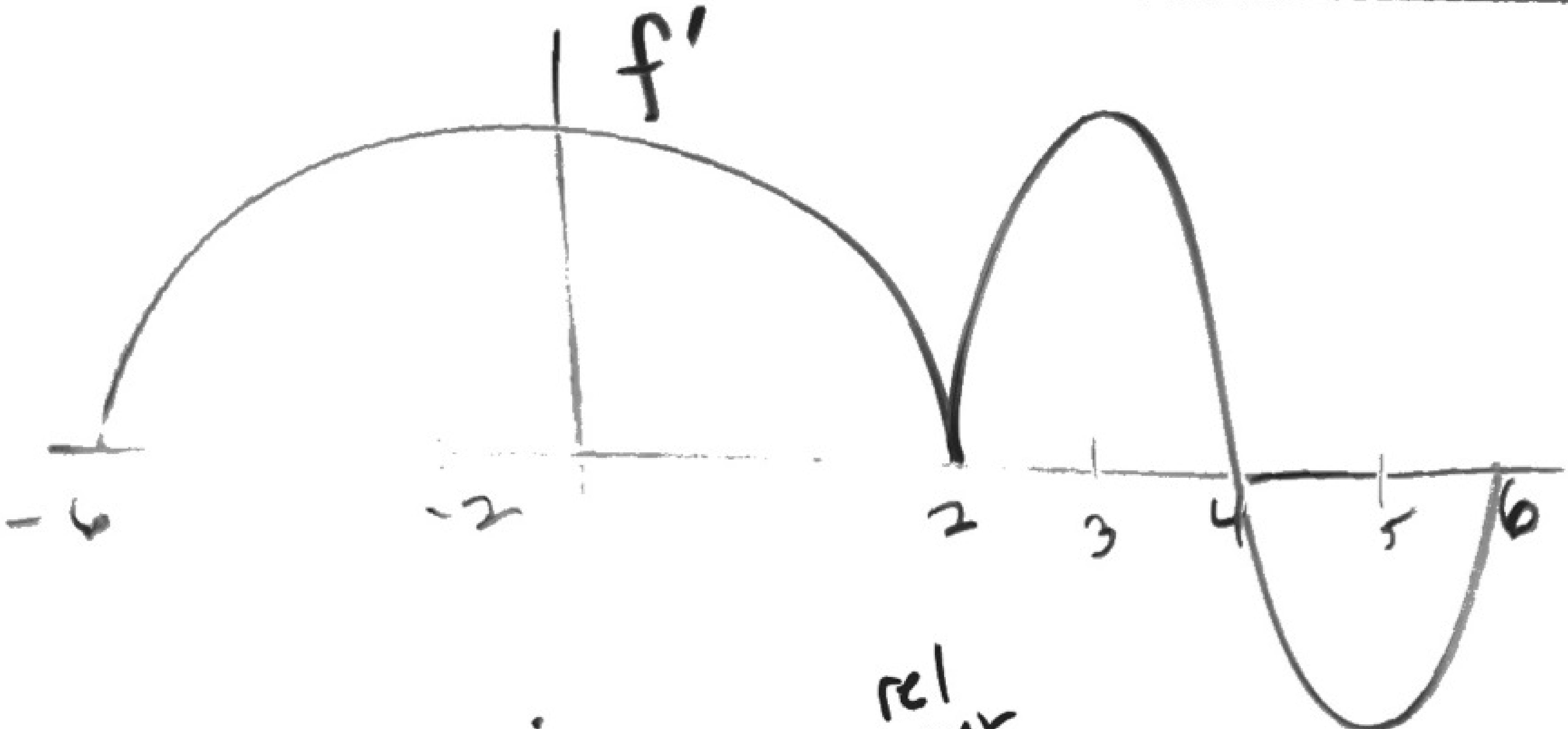


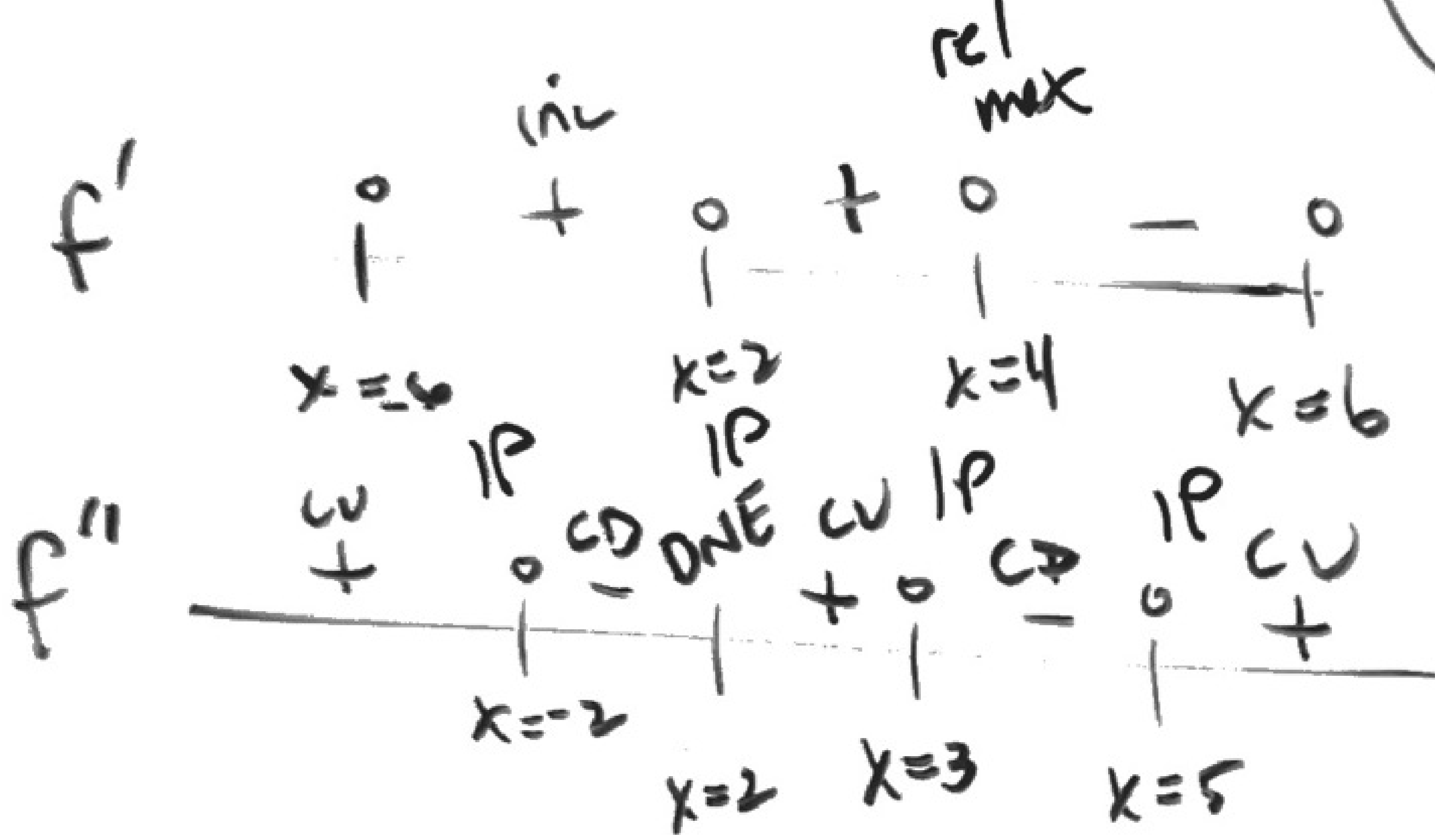
5 Below is a graph of $f(x)$ shown on an interval. The graph of f has a horizontal tangent at c, d, f, and i. In the chart place either a positive sign (+), negative sign (-) or zero (0) at the points a – j for $f(x), f'(x)$ and $f''(x)$. If there is a relative minimum, relative maximum, absolute minimum, absolute maximum or possible inflection point on the interval at these points, put an x in the appropriate column.



Pt	$f(x)$	$f'(x)$	$f''(x)$	Inflection pt.	Relative minimum	Relative maximum	Absolute Minimum	Absolute Maximum
a	-	+	-					
b	0	+	-					
c	+	0	-			x		
d	0	0	+		x			
e	+	+	0	x				
f	+	0	-			x		
g	0	-	-					
h	-	-	0	x				
i	-	0	+		x		x	
j	+	+	+					x

(16)





a) f has a rel. max at $x = 4$ b.c.
f' changes from pos. to neg. here.

b) f has I.P.'s at $x = -2$, $x = 2$, $x = 3$, $x = 5$
b.c. f'' changes sign here.

⑦

$$f(x) = 6x^2 - x^3 - 1$$

$$f'(x) = 12x - 3x^2 \stackrel{?}{=} 0$$

$$3x(4 - x) = 0$$

$$x = 0 \quad x = 4$$

$$f''(x) = 12 - 6x \stackrel{?}{=} 0$$

$$6(2 - x) = 0$$

$$x = 2$$

f': dec $-$ | rel. min 0 at $x=0$ | inc $+$ | rel max 0 at $x=4$ | dec $-$

f'': CU $+$ | I.P. 0 at $x=2$ | CD $-$

- f has a rel. min at $(0, f(0)) = (0, -1)$
  b.c. f' changes from neg. to pos. here.
- f has a rel. max at $(4, f(4)) = (4, 31)$
  b.c. f' changes from pos. to neg. here.
- f has I.P. at $(2, f(2)) = (2, 15)$
  b.c. f'' changes sign here.

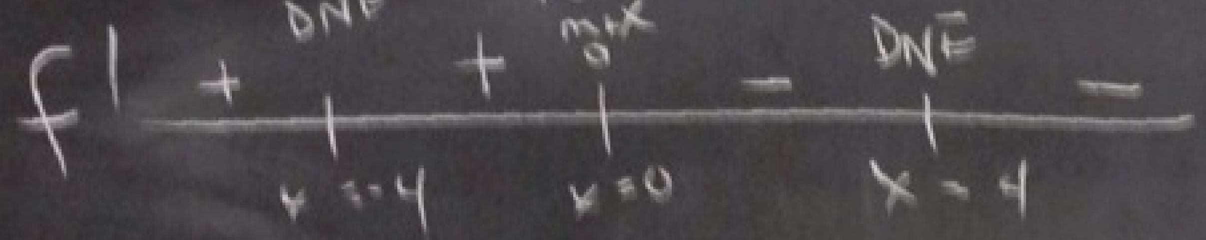
(8) $f(x) = \dfrac{x^2+1}{x^2-16}$

$$f'(x) = \frac{(x^2-16)2x - (x^2+1)(2x)}{(x^2-16)^2}$$

$$= \frac{\cancel{2x^3} - 32x - \cancel{2x^3} - 2x}{(x^2-16)^2}$$

$$= \frac{-34x}{(x^2-16)^2} \overset{?}{=} 0$$

C.P. @ $x=0$, $x=\pm 4$



• $f(x)$ is increasing on $(-\infty,-4),(-4,0)$

b.c. $f'(x) > 0$ on these intervals.

• $f(x)$ is decreasing on $(0,4)$ and $(4,\infty)$

b.c. $f'(x) < 0$ on these intervals.

9. $f(x) = x^3 + 6x^2 + 1$ $\quad [-5, 3]$

$f'(x) = 3x^2 + 12x \overset{?}{=} 0$

$3x(x+4) = 0$

$x = 0, \quad x = -4$

f': $+$ (rel max) at $x=-4$, $-$, (rel min) at $x=0$, $+$

$f(-4) = 33$ <u>rel max</u>

$f(0) = 1$ $\boxed{\text{abs min}}$, <u>rel. min</u>

$f(-5) = 26$

$f(3) = 82$ $\boxed{\text{abs. max}}$

$$\begin{array}{r} 96 \\ -64 \\ \hline 32 \end{array}$$

-125

$$\begin{array}{r} 27 \\ 54 \\ \hline 81 \end{array}$$