

P77

$$v(t) = -9.8t + v_0 \quad a(t) = -9.8$$

When an object is subjected to gravity, its position function is given by $s(t) = -16t^2 + v_0t + s_0$, where t is measured in seconds, $s(t)$ is measured in feet, v_0 is the initial velocity (velocity at $t = 0$) and s_0 is the initial position (position at $t = 0$). The formula is given by $s(t) = -4.9t^2 + v_0t + s_0$ if $s(t)$ is measured in meters.

From our original $s(t) = -16t^2 + v_0t + s_0$, we can calculate the velocity function $v(t) = -32t + v_0$ and the acceleration function $a(t) = -32$. This is the acceleration due to gravity on earth.

When an object is thrown upward, it is subjected to gravity. We are usually interested how high the particle reaches and how fast it is going when it impacts the ground or water. Let us analyze what these mean:

When an object reaches its maximum height, what is its velocity? 0

When an object hits the ground, what is its final position? 0

So to find the maximum height of an object, set $v(t) = 0$, solve for t , and find $s(t)$

So, to find the velocity of an object when it hits the ground, set $s(t) = 0$, solve for t , and find $v(t)$

$$s(t) = -16t^2 + 112t + 0$$

$\begin{matrix} v_0 \\ \downarrow \end{matrix}$
 $\begin{matrix} s_0 \\ \downarrow \end{matrix}$

$$v_0 = 112 \text{ ft/sec}$$

Example 5) A projectile is launched vertically upward from ground level with an initial velocity of 112 ft/sec.

- a. Find the velocity and speed at $t = 3$ and $t = 5$ seconds.

$$v(t) = -32t + 112$$

$$v(3) = 16 \text{ ft/sec}$$

$$v(5) = -48 \text{ ft/sec}$$

$$|v(3)| = 16 \text{ ft/sec}$$

$$|v(5)| = 48 \text{ ft/sec}$$

- b. How high will the projectile rise?

$$v(t) = -32t + 112 \stackrel{?}{=} 0$$

$$t = 3.5 \text{ sec.}$$

$$s(3.5) = 196 \text{ ft}$$

- c. Find the speed of the projectile when it hits the ground.

$$s(t) = -16t^2 + 112t = 0$$

$$-16t(t - 7) = 0$$

$$t = 0 \quad t = 7$$

$$v(7) = -112 \text{ ft/sec}$$

$$\text{speed} = 112 \text{ ft/sec}$$

$$s = -16t^2 + v_0 t + s_0$$

Example 7) A ball is dropped from the top of the Washington Monument which is 555 feet high.

a) How long will it take for the ball to hit the ground?

$$v_0 = 0 \text{ ft/sec}$$

$$s_0 = 555 \text{ ft}$$

$$s(t) = -16t^2 + 555 \stackrel{?}{=} 0$$

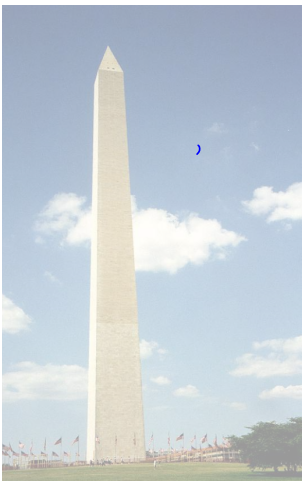
$$t \approx 5.89 \text{ sec}$$

b) Find the ball's speed at impact.

$$v(t) = -32t$$

$$v(5.89) = -188.47 \frac{\text{ft}}{\text{sec}}$$

$$\text{speed} = 188.47 \text{ ft/sec}$$



$$3. s(t) = -t^3 + 9t^2 - 24t + 1$$

$$4. s(t) = t + \frac{9}{t+1} + 1$$

$$v = 1 - \frac{9}{(t+1)^2} = 0$$

$$\frac{9}{(t+1)^2} = 1$$

$$9 = (t+1)^2$$

$$\pm 3 = t+1$$

$$-1 \pm 3 = t$$

$$\text{CP: } \cancel{t = -4} \vee t = 2$$

$$\cancel{t = -1}$$

Warmup: Do this in the same way we did our examples yesterday...

1. The position of a particle moving along a straight line is given by $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$. Perform an analysis of the particle's direction, acceleration, motion, and position.

$$\begin{aligned} v(t) &= 6t^2 - 18t + 12 \\ &= 6(t^2 - 3t + 2) \\ &= 6(t - 2)(t - 1) \end{aligned}$$

$$\begin{aligned} a(t) &= 12t - 18 \\ &= 6(2t - 3) \\ t &= 3/2 \end{aligned}$$

$$\begin{aligned} s(0) &= -4 \\ s(1) &= 1 \\ s(1.5) &= .5 \\ s(2) &= 0 \end{aligned}$$

