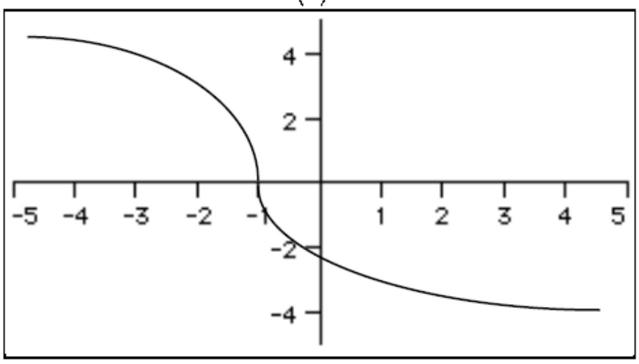
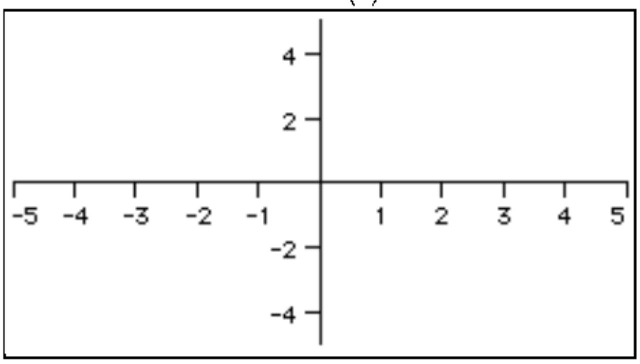
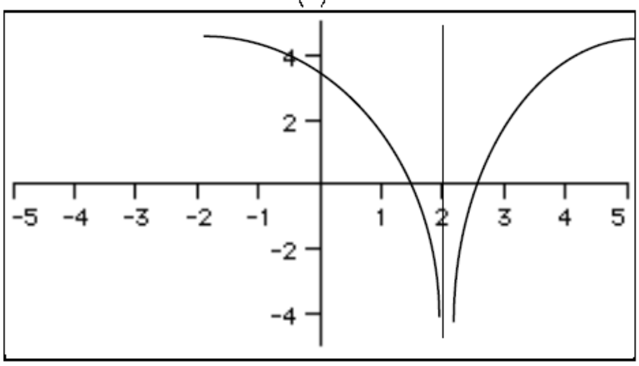
Example 3) You are given a graph of $f'(x)$. Draw a picture of a possible $f(x)$.

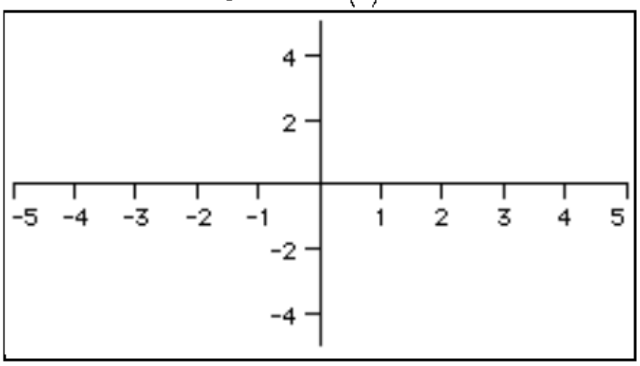
a. $f'(x)$ possible $f(x)$



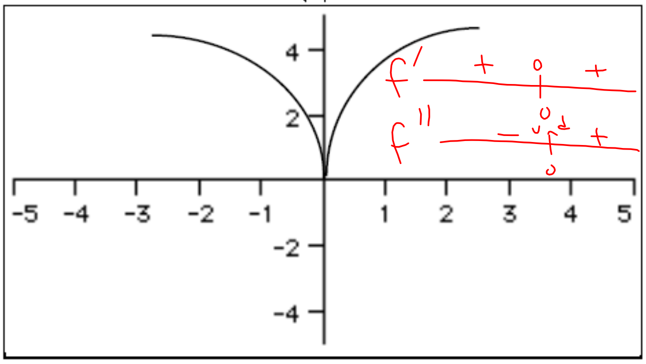


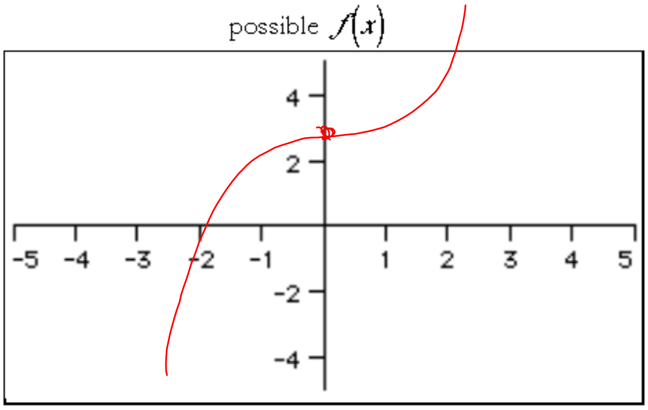
b. $f'(x)$ possible $f(x)$



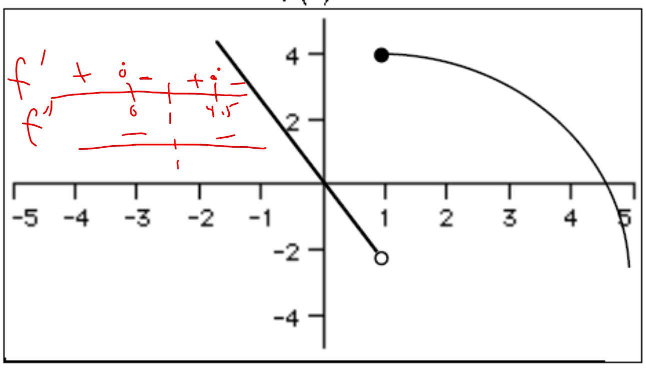


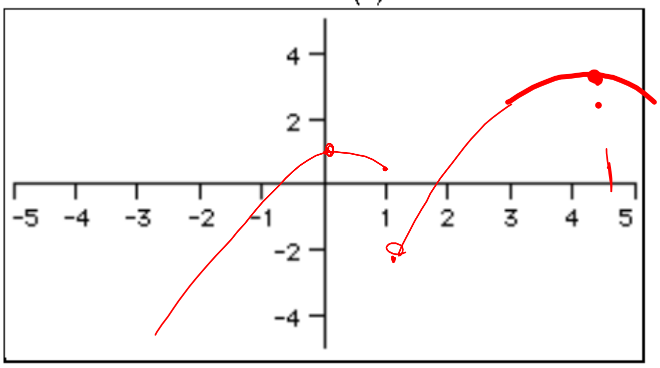
c. $f'(x)$ possible $f(x)$





d. $f'(x)$ possible $f(x)$.



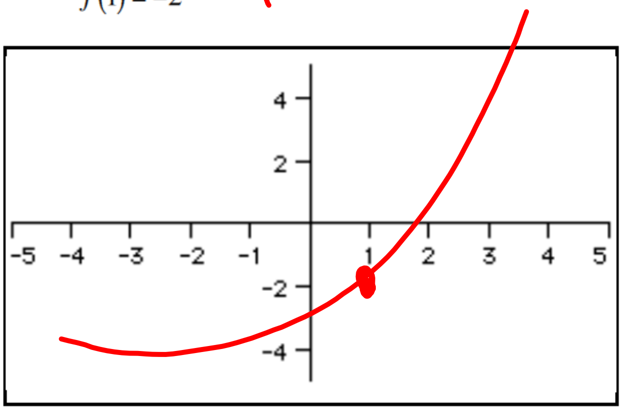


Example 4) Sketch a possible $f(x)$ given the following information.

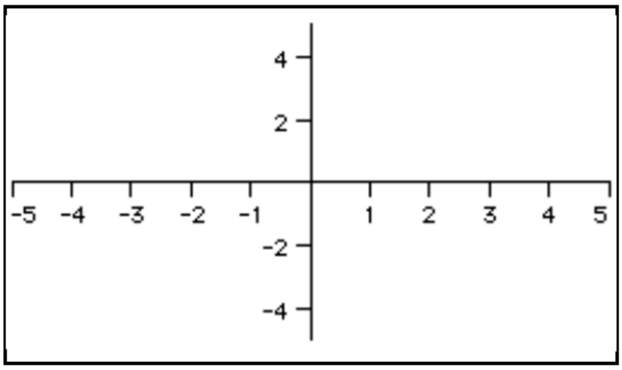
a. $f'(x) > 0$
$f''(x) > 0.$
$f(1) = -2$

f' +

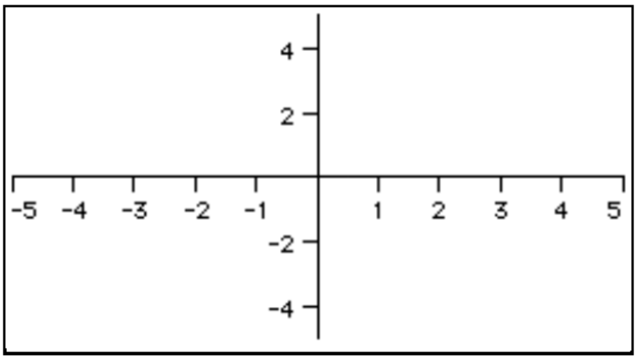
f'' +



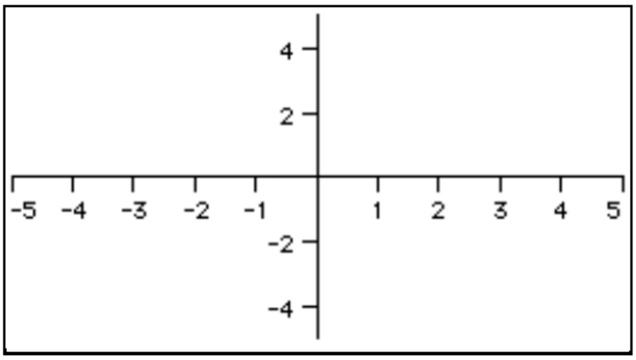
b. $f'(x) > 0, x > 1, \quad f'(x) < 0, x < 1, \quad f'(1) = 0$
$f''(x) > 0, \quad f(1) = -1$



c. $f'(x) > 0, x > 2, \quad f'(x) = 0, x \le 2$
$f''(x) > 0, x > 2 \quad f(2) = 1$



d. $f'(x) > 0, x > -1, \quad f'(x) < 0, x < -1$
$f''(x) < 0, \qquad f(-1) = -4$



e. $f'(x) > 0, x > 1, f'(x) > 0, x < -3, f'(x) < 0, -3 < x < 1$
$f'(-3) = 0, \qquad f'(1) = 0$
$f''(x) < 0, x < 0, \; f''(x) > 0, x > 0$

f. $f'(x) > 0, x > 2, f'(x) = -1, x < 2, f'(2)$ DNE
$f''(x) < 0, x > 2, \; f(2) = 0$

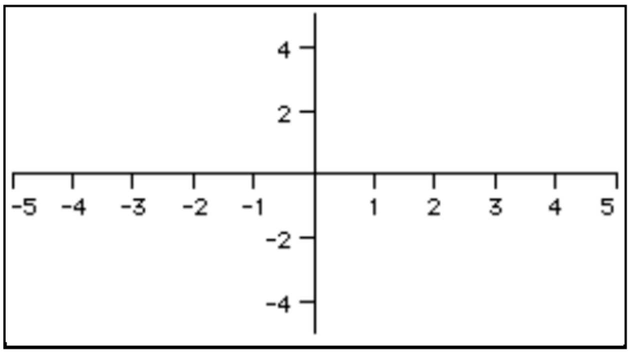
e. $f'(x) > 0, x > 1$, $f'(x) > 0, x < -3$, $f'(x) < 0, -3 < x < 1$

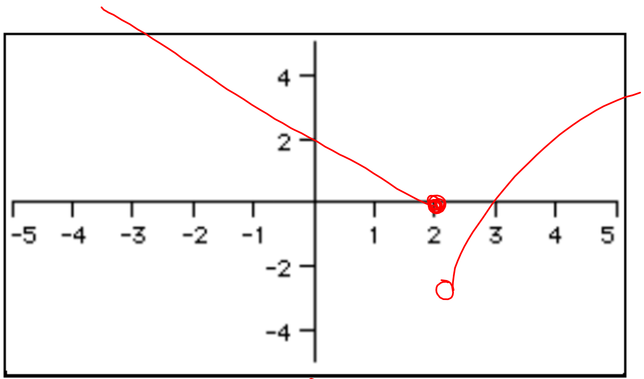
$f'(-3) = 0$, $f'(1) = 0$

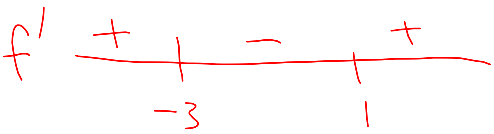
$f''(x) < 0, x < 0$, $f''(x) > 0, x > 0$

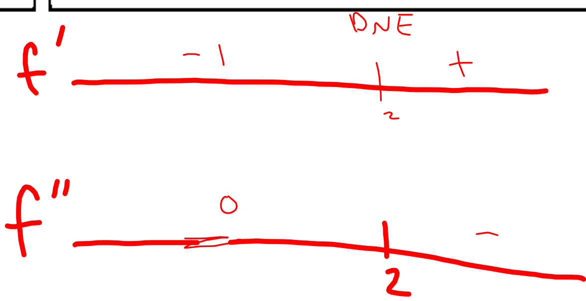
f. $f'(x) > 0, x > 2$, $f'(x) = -1, x < 2$, $f'(2)$ DNE

$f''(x) < 0, x > 2$, $f(2) = 0$





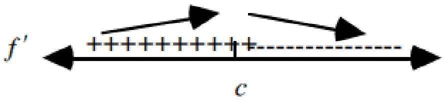




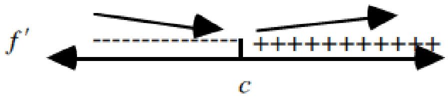
Now that we can determine the graph of a function by examining its first and second derivatives, we now attack the problem from algebraically. We wish to graph some function $f(x)$ by finding its relative maximum and minimum (extrema). With the advent of graphing calculators, a lot of this work is now done by technology and the act of sheer graphing of functions is being reduced on the A.P. exam.

To determine relative extrema of a function , do the following steps.

1. Find the derivative $f'(x)$. It is best that it be in a fraction form.
2. Find the critical values by setting $f'(x) = 0$ or by finding where $f'(x)$ is undefined. If $f'(x)$ is in fraction form, you merely set the numerator and denominator of $f'(x) = 0$ and solve.
3. Make a sign chart for $f'(x)$. Be sure you label it. On it, you will place every critical value c found above.
4. If the sign chart switches from positive to negative at a critical value, at c, it is a relative maximum. If the sign chart switches from negative to positive at a critical value, at c, it is a relative minimum.



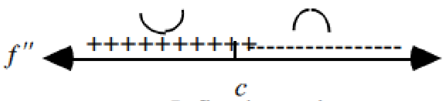
relative maximum



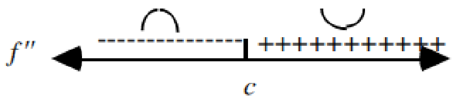
relative minimum

If there is no switch of signs at c, then c is not a relative minimum or maximum.

5. You have identified the x-values where extrema occur. If you are asked to find the **points** where relative maxima and minima occur, you must take every maximum and minimum value c found above and plug them into the function. That is, find $f(c)$.
6. Inflection points occur where the sign of the second derivative $f''(x)$ switches sign. Take the second derivative $f''(x)$, put in fraction form and find every value c where either the numerator or denominator of $f''(x)$ equals zero.
7. If the sign chart switches signs at c then an inflection point occurs at c. We don't care if $f''(c)$ exists.



Inflection point



Inflection point

If there is no switch of signs at c, then c is not a point of inflection.

8. You have identified the x-values where points of inflection occur. If you are asked to find the actual **point of inflection**, you must take every inflection point value c found above and plug it into the function. That is, find $f(c)$.

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Example 5) Find all points of relative maximum and relative minimum and points of inflection if any. Justify your answers. Confirm by calculator.

a) $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x \overset{?}{=} 0$

$= 3x(x-2) = 0$

$x = 0 \quad x = 2$

$f''(x) = 6x - 6 \overset{?}{=} 0$

$6(x-1) = 0$

$x = 1$

b. $f(x) = 4x^3 - x^4$

f': + 0 (rel max) − 2 (rel min) +

f'': − (CD) 1 (IP) + (CU)

Rel. max at $(0, 0)$ b.c. f' changes from + to − there.

Rel min @ $(2, -4)$ b.c. f' changes from − to + there.

I.P. at $(1, -2)$ b.c. f'' changes sign there.

c) $f(x)=6x^5-10x^3$

d. $f(x)=-\cos x$

$f'(x) = 30x^4 - 30x^2 \overset{?}{=} 0$

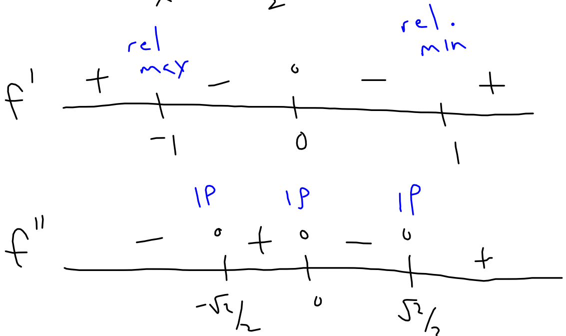
$30x^2(x^2-1) = 0$

$x=0,\ x=\pm 1$

$f''(x) = 120x^3 - 60x = 0$

$60x(2x^2-1) = 0$

$x=0 \quad x = \pm\frac{\sqrt{2}}{2}$ $\sqrt{1/2}$



Rel max @ $(-1, 4)$ b.c. f' changes from + to − here.

Rel min @ $(1, -4)$ b.c. f' changes from − to + here.

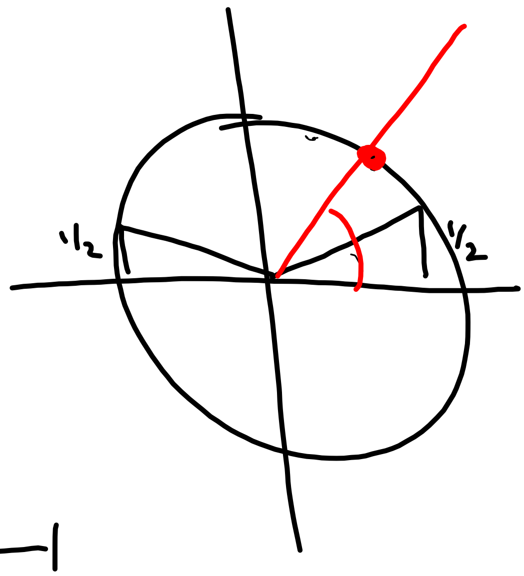
IPs at $(0, 0)$, $(\sqrt{2}/2,\ )$, $(-\sqrt{2}/2,\ )$ b.c. f'' changes sign here.

d. $f(x) = -\cos x - \frac{1}{2}x$ on $[0, 2\pi]$

$f'(x) = +\sin x - \frac{1}{2} \stackrel{?}{=} 0$

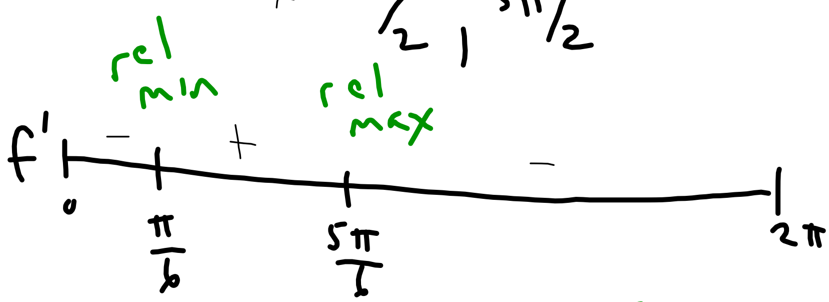
$\sin x = +1/2$

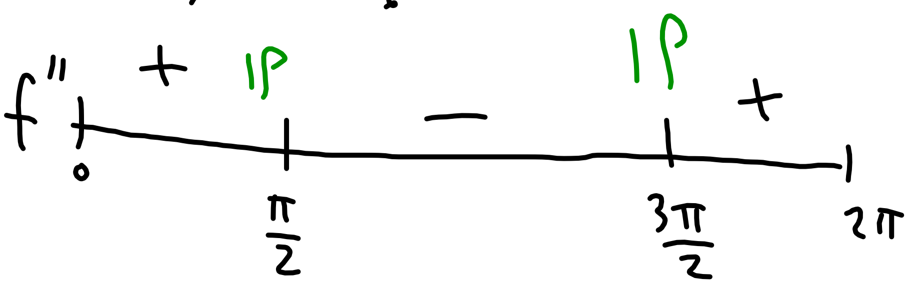
$x = \frac{\pi}{6}, \frac{5\pi}{6}$



$f''(x) = \cos x \stackrel{?}{=} 0$

$x = \pi/2, 3\pi/2$





Rel min @ $\left(\frac{\pi}{6}, -1.128\right)$ b.c. f' ch. from − to + here.

Rel max @ $\left(\frac{5\pi}{6}, -.443\right)$ b.c. f' ch. from + to − here.

I.P.'s @ $\left(\frac{\pi}{2}, -.785\right)$ and $\left(\frac{3\pi}{2}, -2.356\right)$ b.c. f'' ch. sign here.

don't do IP's

e. $f(x) = \dfrac{4}{x^2+4}$

$f'(x) = \dfrac{-4(2x)}{x^2+4} \overset{?}{=} 0$

$-8x = 0$

$x = 0$

f': $+$ | 0 at $x=0$ | $-$

Rel max @ $(0, 1)$ b.c. f' ch. from $+$ to $-$ there.

f. $f(x) = \dfrac{x^2+1}{x^2-9}$

$f'(x) = \dfrac{(x^2-9)(2x)-(x^2+1)(2x)}{(x^2-9)^2}$

$= \dfrac{-18x \quad -2x}{(x^2-9)^2}$

$f'(x) = \dfrac{-20x}{(x^2-9)^2} \overset{?}{=} 0$

c.p.'s at $x=0$ and $x=\pm 3$

f': $+$ | und at -3 | $+$ | 0 at 0 (rel max) | $-$ | und at 3 | $-$

Rel max @ $\left(0, -\frac{1}{9}\right)$ b.c. f' ch. from $+$ to $-$ here.