p 108

Optimization Problems - Classwork

Many times in life we are asked to do an optimization problem - that is, find the largest or smallest value of some quantity that will fufill a need. Typical situations are:

- find the route which will minimize the time it takes me to get to school.
- build a structure using the least amount of material.
- build a structure costing the least amount of money.
- build a yard enclosing the most amount of space.
- find the least medication one should take to help a medical problem.
- find how the most one should charge for a CD in order to make as much money as possible.

Methods for Solving Optimization Problems

1. Assign variables to all given quantities and quantities to be determined. Don't be afraid to use letters you usually do not use (*p,m,g*, etc.). When feasible, make a sketch of the problem.

2. Making a chart of possible answers allows you to see a relationship between variables. While not necessary, it is helpful.

3. Write a "primary" equation for the quantity you found that needs to be maximized or minimized

   Area of Rectangle = length • width

   Distance = rate • time

   Volume of rectangular solid = length • width • height

   Hypotenuse = $\sqrt{x^2 + y^2}$

   Perimeter of rectangle = 2• length + 2• width

   Volume of cylinder = π(radius^2) • height

4. Reduce the right side of this "primary equation" to one having a single variable. If there is more than one variable on the right side, you must write a "secondary" equation (a restriction or constraint) relating the variables of the primary equation.

5. Take the derivative of the equation and **set equal to zero**. If you get more than one answer, make a sign chart to determine whether it represents a maximum or minimum. Pay attention to whether that value makes sense. Time is rarely negative (it can't take negative 7 hours to run a race). You cannot use more than you have (you can't have a length of 8 feet when you only have 6 feet of fencing).

6. **Be sure that you answer the question that is asked.** If you are asked to find a minimum or maximum value of some quantity, you must plug your answer from (4) into your primary equation.

7. If you are to find a maximum or minimum on a closed interval, you must test the endpoints as well. Make sure your work is clear.

8. You can verify your answers by graphing your primary equation with one variable on the calculator. Use your $\boxed{\text{2nd}}$ $\boxed{\text{CALC}}$ maximum or minimum function.

Example 1) Two numbers add up to 40. Find the numbers that maximize their product.

Smaller Number	1	5	10			
Larger Number	39	35	30			
Product	39	175	300			

Primary

Product $= xy$

Product $= x(40-x) = 40x - x^2$

Product$' = 40 - 2x = 0$

$40 = 2x$

$20 = x$

Secondary

$x + y = 40$

$y = 40 - x$

Prod$'$ + rel max −

20

The values that maximize the product are 20 and 20.

Example 2) A rectangle has a perimeter of 71 feet. What length and width should it have so that its area is a maximum? What is this maximum area?

Width						
Length						
Area						

Primary *Secondary*

area = length · width

$71 = 2L + 2w$

area = $l \cdot w$

$w = \frac{71 - 2l}{2}$

area = $l(35.5 - l)$

area' = $35.5 - 2l \stackrel{?}{=} 0$

$71/4 = l$

$4\overline{)71}$ 177, 30

area' + rel max —

$l = 71/4$

The area is maximized when the length and width are both $71/4$ ".

Example 3) Find two positive numbers that minimize the sum of twice the first number plus the second if the product of the two numbers is 288.

First Number						
Second Number						
Sum						

Primary *Secondary*

Let $x = 1^{st}$
$y = 2^{nd}$

$$xy = 288$$
$$y = 288/x$$

$$Sum = 2x + y$$

$$Sum = 2x + \frac{288}{x}$$

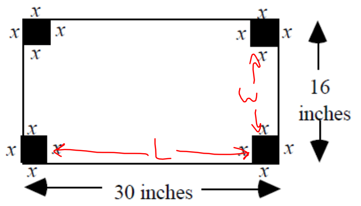
$$Sum' = 2 - \frac{288}{x^2} = 0$$
$$2x^2 = 288$$
$$x^2 = 144$$
$$x = \pm 12$$

Sum': $+$ (rel max) -12 $-$ 0 $-$ (rel min) 12 $+$

1^{st} number is 12, second is 24!

Example 4) An open box is to be made from a piece of metal 16 by 30 inches by cutting out squares of equal size from the corners and bending up the sides. What size square should be cut out to create a box with greatest volume. What is the maximum volume as well?

Primary



$L = 30 - 2x$

$W = 16 - 2x$

$V = L \cdot W \cdot H$

$V = (30 - 2x)(16 - 2x)\,x$

$V = 4x(15 - x)(8 - x)$

$V = 4x(120 - 23x + x^2)$

$V = 4(120x - 23x^2 + x^3)$

$V' = 4 \cdot (120 - 46x + 3x^2) = 0$

$(3x - 10)(x - 12) = 0$

$x = 10/3$ $\quad$ $x = 12$

x can't be 12 in this problem. Do you know why?

V': $+$ | $-$ at $10/3$ (rel max)

The volume is maximized when the square cut outs have sides of length $10/3''$.

The volume will be approx. 4061.1 in^3.

$V = \left(30 - \frac{5}{3}\right)\left(16 - \frac{5}{3}\right)\frac{10}{3}$

$= \frac{85}{3} \cdot \frac{43}{3} \cdot \frac{10}{3}$

$= 4061.11$