

Economic Optimization Problems - Classwork

Example 1) A trucking company has determined that the cost per hour to operate a single truck is given by $C(s) = 0.0001s^2 - 0.01s + 112$ where s is the speed that the truck travels. At what speed is the total cost per hour a minimum? What is the hourly cost to operate the truck?

$$\text{minimize: } C(s) = .0001s^2 - .01s + 112$$

$$C' = .0002s - .01 \stackrel{?}{=} 0$$

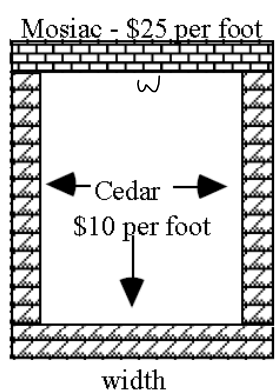
$$.0002s = .01$$

$$C' \begin{array}{c} \text{rel.} \\ \text{min.} \end{array} \begin{array}{c} - \quad | \quad + \\ 50 \end{array}$$

$$s = \frac{.01}{.0002} = 50$$

The speed when C is minimized is 50 mph.
 $C(50) = 11.75$

Example 2) A nursery wants to add a 1,000-square-foot rectangular area to its greenhouse to sell seedlings. For aesthetic reasons, they have decided to border the area on three sides by cedar siding at a cost of \$10 per foot. The remaining side is to be a wall with a brick mosaic that costs \$25 per foot. What should the dimensions of the sides be so that the cost of the project will be minimized?



$$\text{cost} = \text{mosaic} + \text{cedar}$$

$$\text{cost} = 25w + 10(2l + w); \quad l = \frac{1000}{w}$$

$$\text{cost} = 25w + 10\left(\frac{2000}{w} + w\right)$$

$$\text{cost} = 35w + \frac{20000}{w}$$

Width	Length	Cedar Cost	Mosaic Cost	Total Cost

$$\text{cost}' = 35 - \frac{20000}{w^2} = 0$$

$$35w^2 = 20000$$

$$w \approx 23.905 \text{ ft}$$

$$l \approx 41.833 \text{ ft}$$

$$\text{cost} = \$1673.32$$

Example 3) A real estate company owns 100 apartments in New York City. At \$1,000 per month, each apartment can be rented. However, for each \$50 increase, there will be two additional vacancies. How much should the real estate company charge for rent to maximize its revenues?

\$50 increases	Rent	Apts. rented	Revenue
0	1000	100	100000
1	1050	98	$(1050)(98) = 102900$
2	1100	96	$= 105600$
3	1150	94	$= 108100$
4			
50			
x	$1000 + 50x$	$100 - 2x$	$(1000 + 50x)(100 - 2x)$

max
=

$$Rev = (1000 + 50x)(100 - 2x)$$

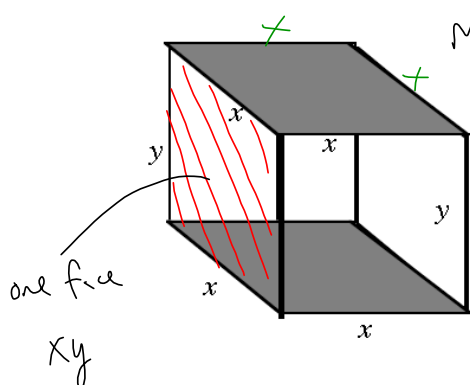
$$Rev = 10000 + 3000x - 100x^2$$

$$Rev' = 3000 - 200x = 0$$

$$x = 15$$

$$Rent = \$1750$$

Example 4) A closed box with a square base is to have a volume of $1,800 \text{ in}^3$. The material for the top and bottom of the box costs \$3 per square inch while the material for the sides cost \$1 per square inch. Find the dimensions of the box that will lead to the minimum total cost. What is the minimum total cost?



$$\text{Min cost} = \$ \text{ sides} + \$ \text{ top/bot}$$

$$\text{cost} = \$1 \cdot (4xy) + \$3 \cdot (2x^2)$$

$$\text{cost} = 4x \cdot \frac{1800}{x^2} + 6x^2$$

$$\text{cost}' = -\frac{7200}{x^2} + 12x = 0$$

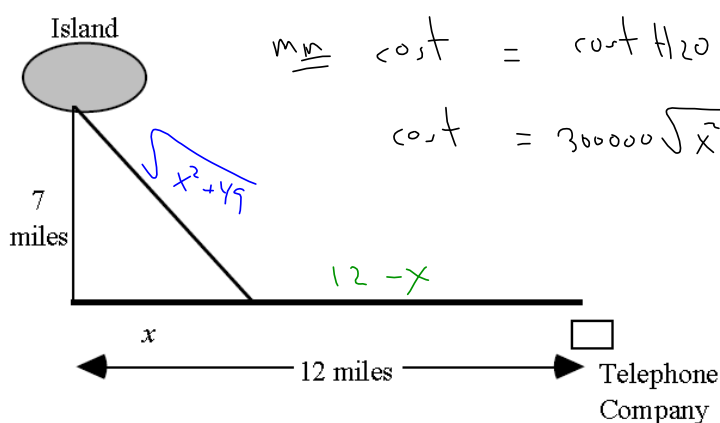
$$x = \sqrt[3]{600} \approx 8.434 \text{ in.}$$

$$\text{base} = 8.434 \text{ in.}$$

$$\text{height} = 25.303 \text{ in.}$$

$$\text{cost} = \$1,280.48$$

Example 5. A telephone wire is to be laid from the telephone company to an island 7 miles off shore at a cost of \$200,000 per mile along the shoreline and \$300,000 per mile under the sea. How should the wire be laid at the least expensive cost if the distance along the shoreline is 12 miles. What is that cost?



$$\min \text{ cost} = \text{cost land} + \text{cost land}$$

$$\text{cost} = 300000 \sqrt{x^2 + 49} + 200000(12 - x)$$

$$\text{cost}' = 300000 \cdot \frac{1}{2} (x^2 + 49)^{-1/2} \cdot 2x - 200000 = 0$$

$$\frac{x}{\sqrt{x^2 + 49}} = \frac{2}{3}$$

$$\frac{x^2}{x^2 + 49} = \frac{4}{9}$$

$$9x^2 = 4x^2 + 196$$

$$5x^2 = 196$$

$$x^2 = 39.2$$

$$x = \sqrt{39.2} \approx 6.261$$

The minimum cost occurs when you run the cable to a point approximately 6.261 mi down the shoreline.

$$\text{Cost} = \$3,965,247.58$$

$$\text{cost}' \underset{6.261}{\overset{\text{rel min}}{=}} 0$$

Example 6) A small television company estimates that the cost (in dollars) of producing x units of a certain product is given by $800 + .04x + .00002x^2$. Find the production level that minimizes the average cost per unit.

Units	Cost	Average Cost
100		
1,000		
5,000		
10,000		

$$\min \text{ average} = \frac{800 + .04x + .00002x^2}{x}$$

$$\text{average} = \frac{800}{x} + .04 + .00002x$$

$$\text{average}' = -\frac{800}{x^2} + .00002 \stackrel{?}{=} 0$$

$$.00002x^2 = 800$$

$$x = 6325 \text{ units}$$

Cost is minimized when they produce 6325 TVs.

$$\text{Cost}' \begin{array}{c} \text{rel.} \\ \text{min.} \end{array} \begin{array}{c} - \\ | \\ + \end{array} \begin{array}{c} 6325 \end{array}$$

Economic Optimization Problems - Homework

1. The profit for Ace Advertising Co. is $P = 230 + 20s - \frac{1}{2}s^2$ where s is the amount (in hundreds of dollars) spent on advertising. What amount of advertising gives the maximum profit?

2. North American Van Lines calculates charges for delivery according to the following rules.

$$\text{Fuel cost} = \frac{v^2}{120} \text{ per hour}$$

$$\text{Driver cost} = \$30 \text{ per hour}$$

Find the speed v that a truck should travel in order to minimize costs for a trip of 120 miles. *Hint: remember that rate • time = distance. Make a chart of possible speeds v and total costs.*

3. Normally a pear tree will produce 30 bushels of pears per tree when 20 (or fewer) pear trees are planted per acre. However, for each additional pear tree planted above 20 trees per acre, the yield per tree will fall by one bushel per tree (why?). How many trees should be planted per acre to maximize the total yield? *Hint: Make a chart like the Apartment Housing sample problem.*

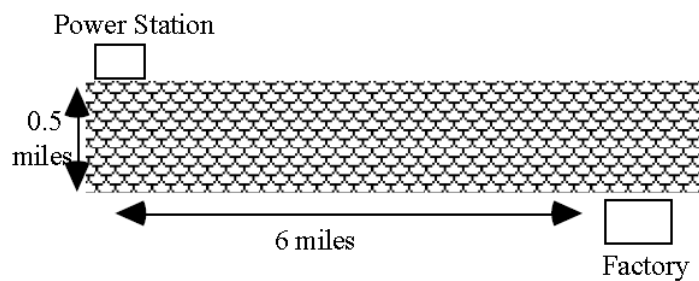
4. Midas Muffler charges \$28 to replace a muffler. At this rate, the company replaces 75,000 mufflers per week nationally. For each additional dollar that the company charges, it tends to lose 1,000 customers a week. For each dollar the company subtracts from the \$28, the company gains 1,000 per week. How much should Midas charge to change a muffler in order to maximize their revenue? What would that revenue be? *Hint: Make a chart like the Apartment Housing sample problem.*

5. A concert promoter knows that 5,000 people will attend an event with tickets set at \$50. For each dollar less in ticket price, an additional 1,000 tickets will be sold. What should the price of a ticket be in order to maximize the total receipts. *Hint: Make a chart like the Apartment Housing sample problem.*

6. A travel agent is offering charter holidays in the Bahamas for college students. For groups of size up to 100, the fare is \$1,000 per student. For larger groups, the fare per person decreases by \$5 for each additional person in excess of 100. Find the size of the group that will maximize the travel agent's revenues. *Hint: Make a chart like the Apartment Housing sample problem.*

7. A real estate office handles 50 apartment units. When the rent is \$540 per month, all units are occupied. However, on the average, for each \$30 increase in rent, one unit becomes vacant. Each occupied unit requires an average of \$36 per month for service and repairs. What rent should be charged to realize the most profit?

8. A power station is on one side of a river that is .5 mile wide, and a factory is 6 miles downstream on the other side. It costs \$6,000 per mile to run power lines overland and \$8,000 per mile to run them underwater. Find the most economical path to lay transmission lines from the station to the factory.



9. A rectangular area is to be fenced in using two types of fencing. The front and back uses fencing costing \$5 a foot while the sides uses fencing costing \$4 a foot. If the area of the rectangle must contain 500 square feet, what should be the dimensions of the rectangle in order to keep the cost a minimum?

10. The same rectangular area is to be built, but now the builder has only \$800 to spend. What is the *largest area* that can be fenced in using the same two types of fencing mentioned above.