① Maximize product $= xy^2$, $x + y = 9$ 128 days until the

$y = 9 - x$

$$\text{product} = x(9-x)^2 = x(81 - 18x + x^2) = x^3 - 18x^2 + 81x$$

$$\text{product}' = 3x^2 - 36x + 81 \stackrel{?}{=} 0$$

$$3(x^2 - 12x + 27) = 0$$

$$3(x-9)(x-3) = 0$$

$x = 9$, $x = 3$

rel max at 3, rel min at 9

product': $+$ (before 3), $-$ (between 3 and 9), $+$ (after 9)

Product is a maximum when $x = 3$ and $y = 6$.

② Maximize Volume $= l \cdot w \cdot h$

$$\text{Volume} = x(24-2x)^2$$
$$= 4x(12-x)^2$$
$$= 4x(144-24x+x^2)$$
$$= 4(x^3-24x^2+144x)$$

$$\text{Volume}' = 4\cdot(3x^2-48x+144) = 0$$
$$= 4\cdot 3(x^2-16x+48)$$
$$= 12(x-4)(x-12)$$

$x=4,\ x=12$

Vol'

		rel max		rel min	
	+	0	−	0	+
		4		12	

You should cut
4" squares
from each corner.

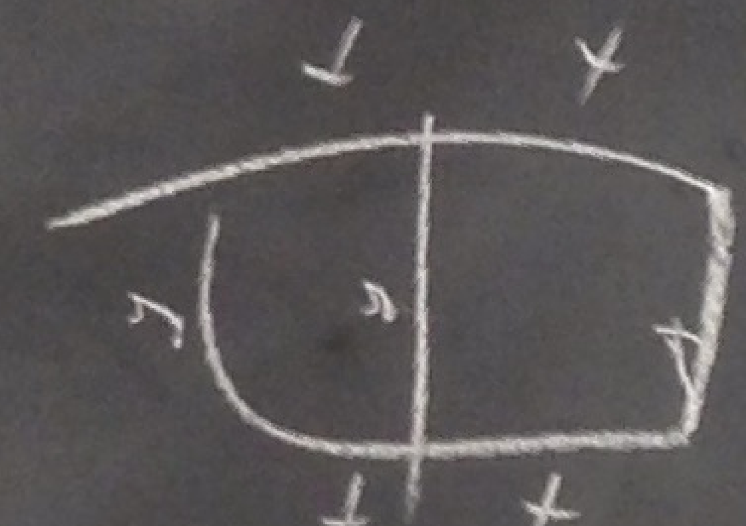
④ Maximize $\text{Area} = 2xy$; $4x + 3y = 200$

$$3y = 200 - 4x$$
$$y = \frac{4}{3}(50 - x)$$

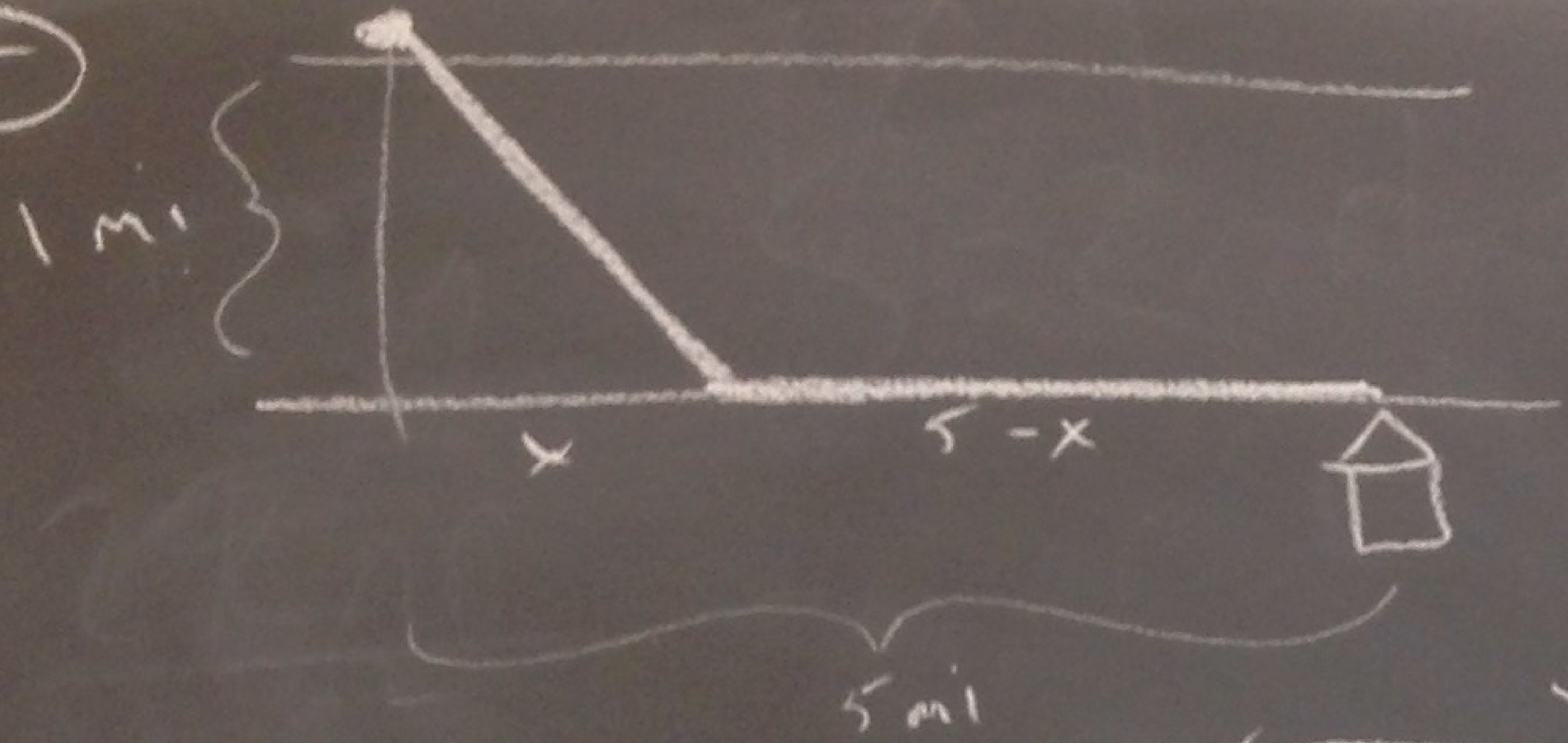
$$\text{Area} = 2x \cdot \frac{4}{3}(50 - x)$$
$$= \frac{8}{3}(50x - x^2)$$
$$\text{Area}' = \frac{8}{3}(50 - 2x) = 0$$
$$x = 25$$

$$\text{Area}' \quad \overset{+}{\longrightarrow} \underset{25}{\overset{\text{max}}{|}} \overset{-}{\longrightarrow}$$

The dimensions of the pen will have $x = 25$ ft, $y = \frac{100}{3}$ ft.



5



$$\min \ \text{Cost} = 3000\left(\sqrt{1+x^2}\right) + 1000(5-x)$$

$$\text{Cost}' = 3000 \cdot \frac{1}{\cancel{2}}\left(1+x^2\right)^{-1/2} \cdot \cancel{2}x - 1000 \overset{?}{=} 0$$

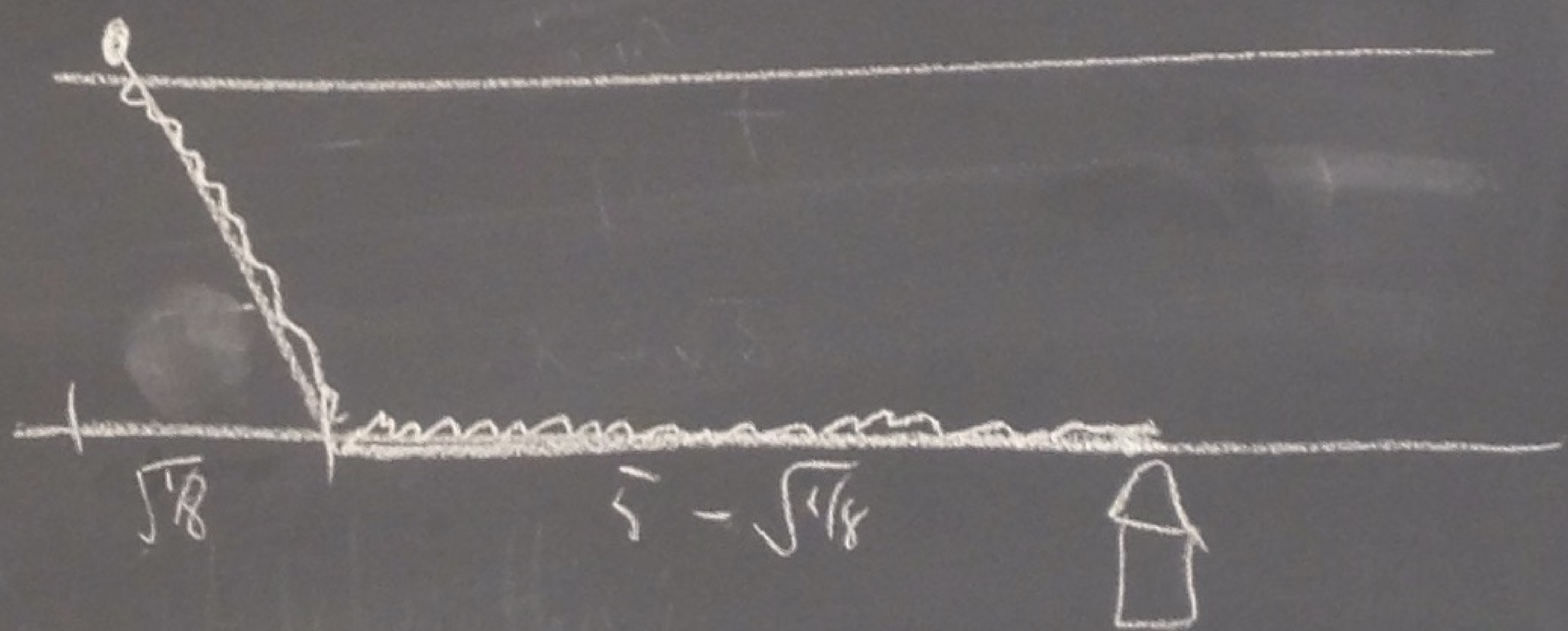
$$\frac{x}{\sqrt{1+x^2}} = \frac{1}{3}$$

$$9x^2 = 1+x^2$$

$$8x^2 = 1$$

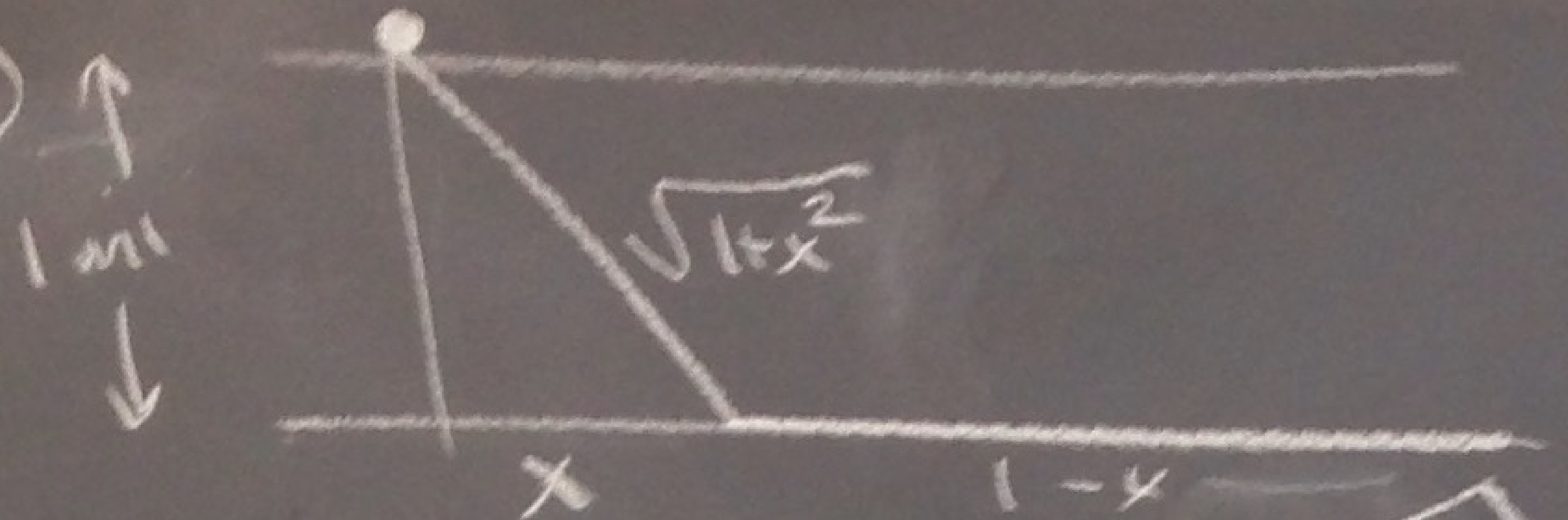
$$x = \sqrt{1/8}$$

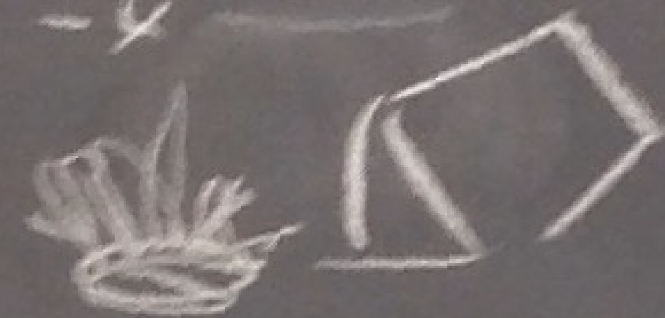
Run it like this:



$$\text{Cost} = \underline{\$7828.43}$$

(6)





min. time $= \dfrac{\sqrt{1+x^2}}{2} + \dfrac{1-x}{3}$

$t = \dfrac{d}{r}$

time$' = \dfrac{1}{\cancel{2}}\cdot\dfrac{1}{2}(1+x^2)^{-1/2}\cdot \cancel{2}x - \dfrac{1}{3} = 0$

$$\frac{x}{\sqrt{1+x^2}} = \frac{2}{3}$$

$$\frac{x^2}{1+x^2} = \frac{4}{9}$$

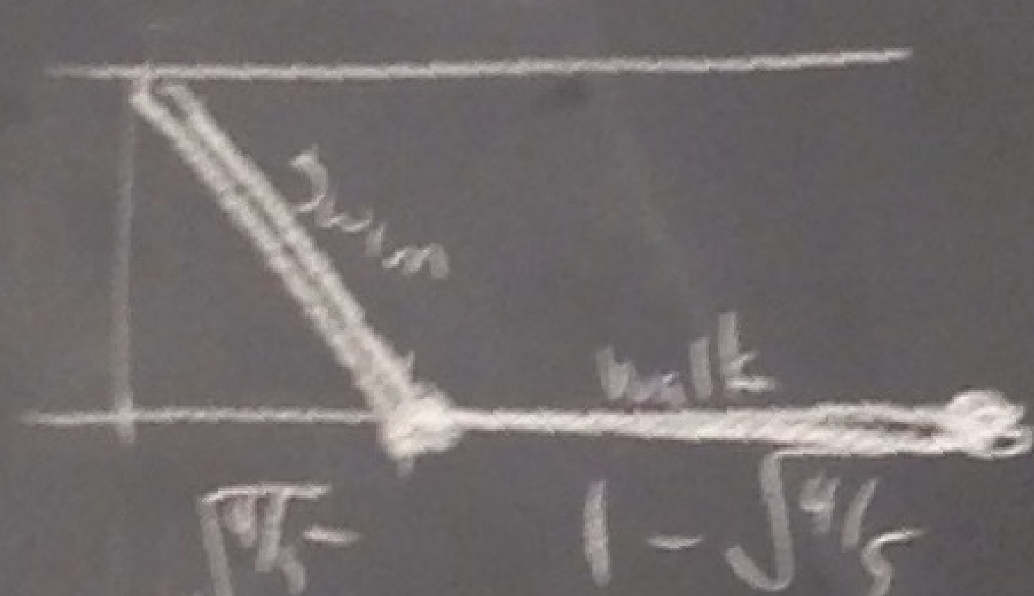
$$9x^2 = 4 + 4x^2$$

$$5x^2 = 4$$

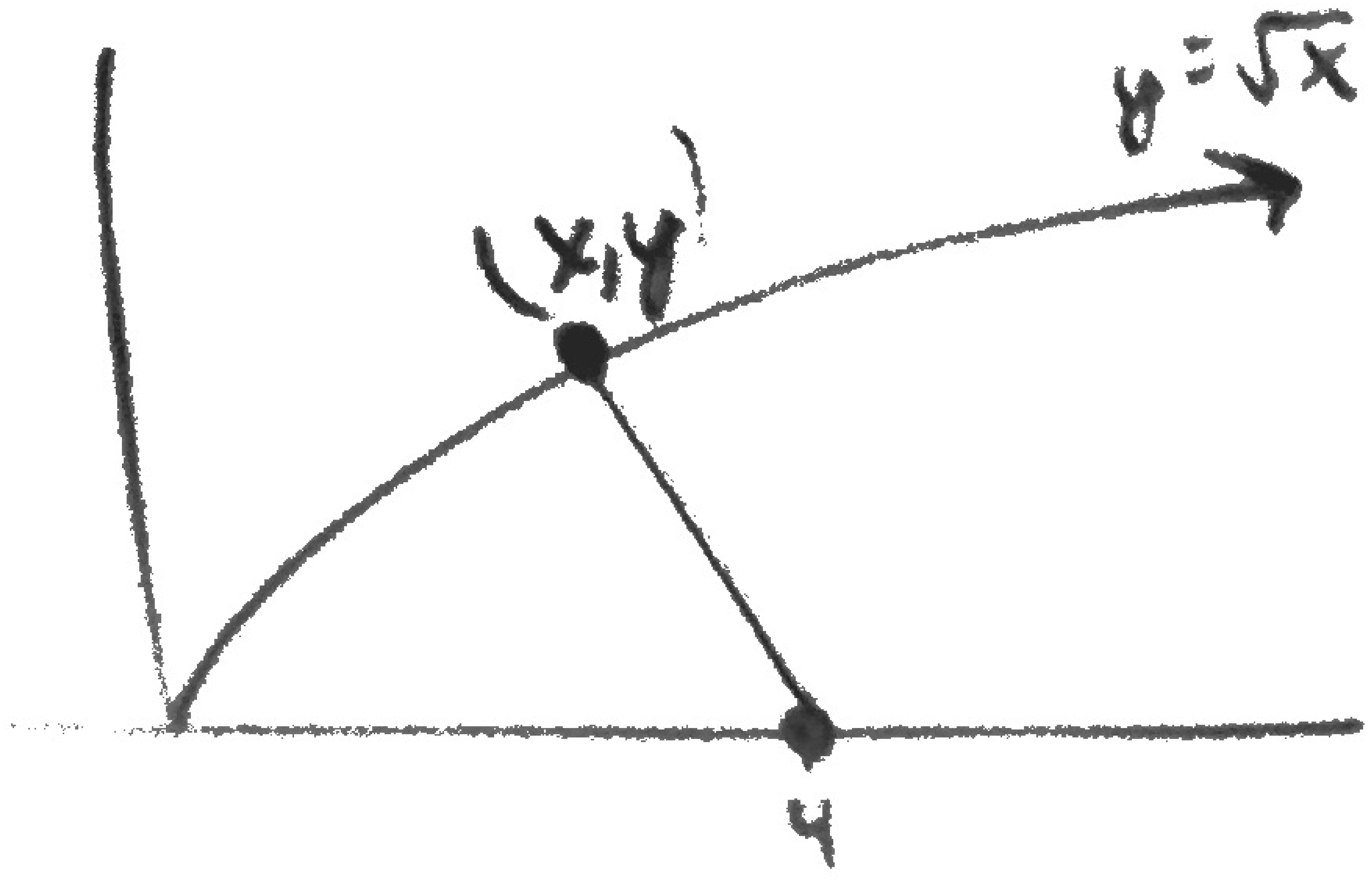
$$x^2 = 4/5$$

$$x = \sqrt{4/5}$$

Go like this to minimize time:



7



min distance $= \sqrt{(x-4)^2 + y^2}$

$= \sqrt{(x-4)^2 + x}$

$= \sqrt{x^2 - 7x + 16}$

dist$' = \frac{1}{2}\left(x^2 - 7x + 16\right)^{-1/2} \cdot (2x - 7) = 0$

$2x - 7 = 0$

$x = 3.5$

dist$'$ — | + (at 3.5)

distance is minimized at the point $\left(3.5, \sqrt{3.5}\right)$.

8

Max Output $= (50 + x)(800 - 10x)$

Output$' = (50 + x) \cdot -10 + (800 - 10x)$

$= -500 - 10x + 800 - 10x$

$= 300 - 20x \stackrel{?}{=} 0$

$300 = 20x$

$15 = x$

Output is maximized when you add 15 trees to the orchard.