p130

Sigma Notation - Classwork

We will switch gears for a section and learn a completely different type of problem. It will be apparent within a few sections why we are seemingly learning something unrelated to integration.

Suppose you were asked to find the sum of the first 5 terms of the following sequence:

1 + 2 + 4 + … = ________ How did you arrive at the answer? ______________________________

The problem with writing such addition problems with the ellipsis (…) , is that the rule for each term is not apparent. We introduce notation called **sigma notation** for such problems using the Greek letter sigma $\sum$.

The sum of n terms $a_1 + a_2 + a_3 + \cdots + a_n$ is written as $\sum_{i=1}^{n} a_i$ where i is the index of summation and a_i is the ith term of the sum. Note that sigma notation does not help you to calculate the sum, only to write the sum.

Examples - Find the following sums.

1) $\sum_{i=1}^{8} 3 = 3+3+3+3\cdots$

$= 24$

2. $\sum_{i=1}^{6} i =$

$= 1+2+3+4+5+6$

$= 21$

3. $\sum_{j=1}^{7} j^2 =$

$= 1+4+9+16+25+36+49$

$=$

4. $\sum_{k=-2}^{3} k^3 = \cancel{-8} + \cancel{-1} + 0 + \cancel{1} + \cancel{8} + 27$

27

7(8)

Since $\sum_{i=1}^{n} a_i$ represents a summation of numbers, we can apply basic properties of addition and multiplication.

$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$ (meaning you can factor out k) $\sum_{i=1}^{n}\left[a_i \pm b_i\right] = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$ (write one sum as 2 sums)

$\begin{array}{r} 6 \\ 28 \\ \times 8 \\ \hline 224 \end{array}$

Find the following sums (calculators allowed)

5) $\sum_{i=1}^{7} 8i = 8+16+24+32+40+48+56$

$= 224$

6) $\sum_{i=1}^{5}\left[5i-2\right] = 3+8+13+18+23$

$= 65$

7) $\sum_{i=1}^{5}\left[(i+2)(i+3)\right]$

$= \sum\left(i^2+5i+6\right)$

$= \sum i^2 + 5\sum i + \sum 6$

8) $\sum_{i=1}^{8} \frac{i}{3}$

9) $\sum_{i=1}^{6} \frac{i-2}{i}$

$\sum_{i=1}^{6} 1 - 2\sum_{i=1}^{6}\frac{1}{i}$

10) $\sum_{i=2}^{8} \sqrt{i^2-1}$

11) $\sum_{i=1}^{100} (-1)^i$

12) $\sum_{i=0}^{10} (-1)^i i^2$

*

Technology: You can use your TI-84 to generate and add terms of these sequences. To create a sequence you will use the SEQ command found in $\boxed{\text{2nd}}$ $\boxed{\text{LIST}}$ OPS. The format of this command is Seq(formula in x, x, starting x, ending x). For instance, problem 5 above $\sum_{i=1}^{7} 8i$ would be Seq(8X,X,1,7). This will generate the sequence. Now to add the terms, you use the sum command found in $\boxed{\text{2nd}}$ $\boxed{\text{LIST}}$ MATH. Use Sum(Ans). You can do this in one fell swoop: Sum(Seq(8X,X,1,7)). You may only sum up to 999 terms.

$$\sum_{i=1}^{7} 8i = 8\sum_{i=1}^{7} i$$

$$= 8\cdot(1+2+3+4+5+6+7)$$

$$= 8\cdot 28$$

$$\cancel{= 244}$$

$$= 224$$

$$\sum_{i=1}^{7} 8i = 8\sum_{i=1}^{7} i$$

$$= 8 \cdot \frac{7 \cdot 8}{2}$$

$$= 224$$

$$\sum_{i=1}^{5}\left[5i-2\right] = 5\sum_{i=1}^{5} i - \sum_{i=1}^{5} 2$$

$$= 5 \cdot \frac{5 \cdot 6}{2} - 10$$

$$= 75 - 10$$

$$= 65$$

As quick as you can, find the sum $\sum_{i=1}^{15} i = $ 120

$$\begin{array}{r} \overset{4}{1}5 \\ \times\ 8 \\ \hline 120 \end{array}$$

Suppose you were asked to find the sum $\sum_{i=1}^{100} i$. Would you add all 100 terms? There must be an easier way.

$$\sum_{i=1}^{100} i = 1 + 2 + 3 + \ldots + 50 + 51 + \ldots + 98 + 99 + 100$$

Instead of adding your terms in sequential order, add the first plus the last, 2nd and next to last, etc. Each gives a sum of 101. Altogether you have 101 added 50 times or 50(101) = 5050.

There are formulas you can use to add many terms. While it is not necessary that you memorize the formulas, you will find them extremely useful for difficult summations.

$$50 \cdot 101 = \frac{100}{2} \cdot 101$$

$\sum_{i=1}^{n} c = cn$	$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Examples - Find the following sums.

13) $\sum_{i=1}^{45} 3i = 3\sum_{i=1}^{45} i$

$= 3 \cdot \frac{45 \cdot 46}{2}$

$=$

14) $\sum_{i=1}^{30} i^2 = \frac{30 \cdot 31 \cdot 61}{6}$

$=$

15) $\boxed{\sum_{i=1}^{50} (2i^3 - 1)}$

$\sum_{i=3}^{45} i = \sum_{i=1}^{45} i - \sum_{i=1}^{2} i$

$= \frac{45 \cdot 46}{2} - \frac{2 \cdot 3}{2}$

$$\sum_{i=1}^{50} 2i^2 - 1 = 2\sum_{i=1}^{50} i^2 - \sum_{i=1}^{50} 1$$

$$= 2 \cdot \frac{50 \cdot 51 \cdot 101}{6} - 50$$

$$= 85800$$

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$