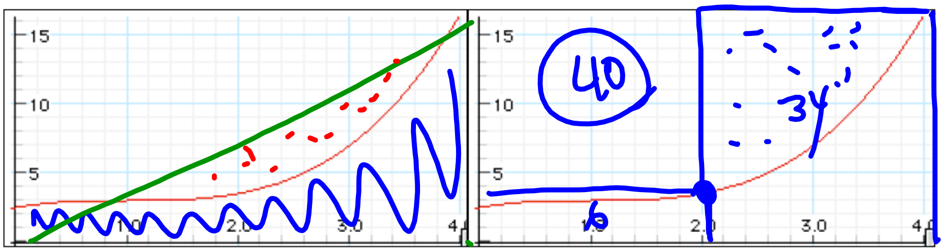
Area Under Curve - Classwork

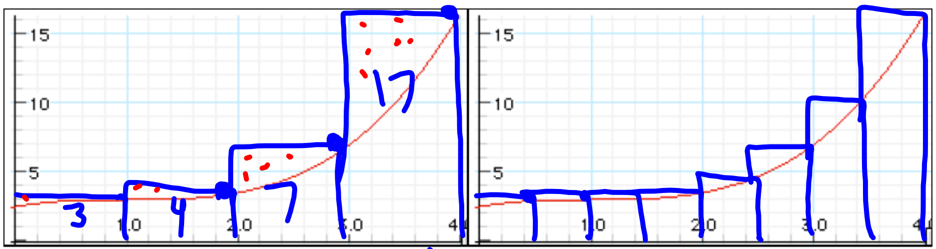
One of the basic problems of calculus is to find the slope of the tangent line (i.e. the derivative) at any point on the curve. The other basic problem is to find the area under the curve, that is the area between the curve and the x-axis between any two values of x.

Below you are given a curve $y = f(x)$. Estimate what you think the area is. Then on the next three graphs, draw 2 rectangles, 4 rectangles, and 8 rectangles, total the areas of each and sum them for another estimate of the total area under the curve.

10
105
125
80
82
25



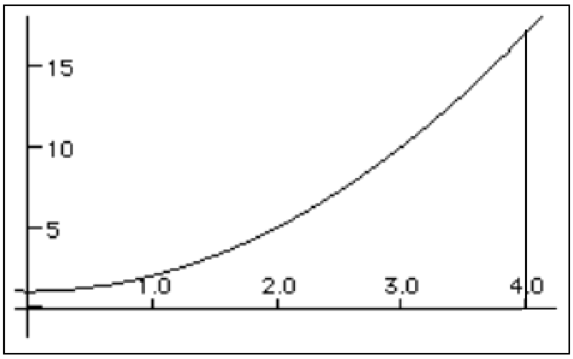
Estimate of area ______________ 2 Rectangles: _______ + ________ = ______



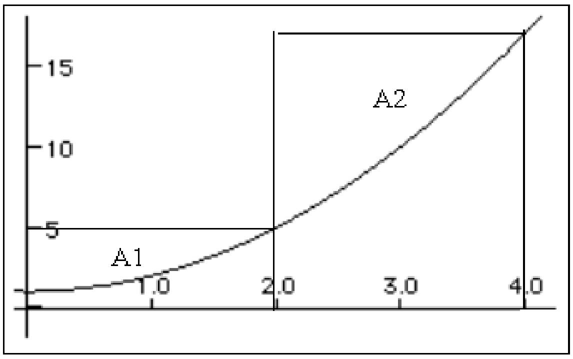
4 Rectangles: ____ + ____ + ____ + ____ = 31 8 Rectangles: ____ + ____ + ____ + ____ + ____ + ____ + ____ + ____ = ______

$= \frac{1}{2}(3+3+3+3+4+6+10+17)$

$= 19.5$

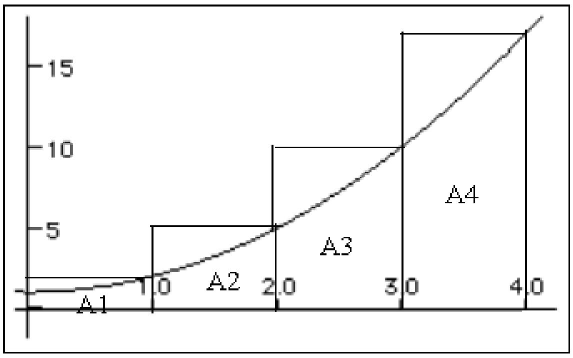


Estimate the area ____________________



Area $\approx A1 + A2 = b_1h_1 + b_2h_2$

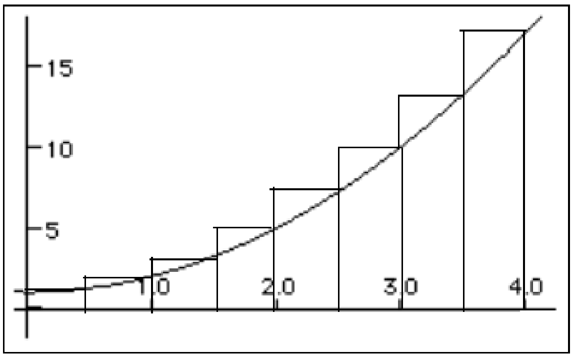
Area $\approx b_1 f(2) + b_2 f(4) =$



Area $\approx A1 + A2 + A3 + A4$

Area $\approx b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4$

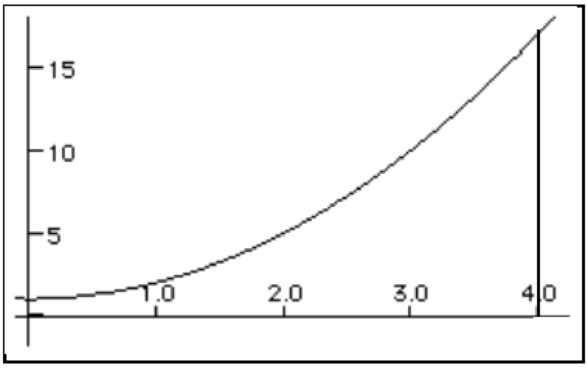
Area $\approx b_1 f(1) + b_2 f(2) + b_3 f(3) + b_4 f(4) =$



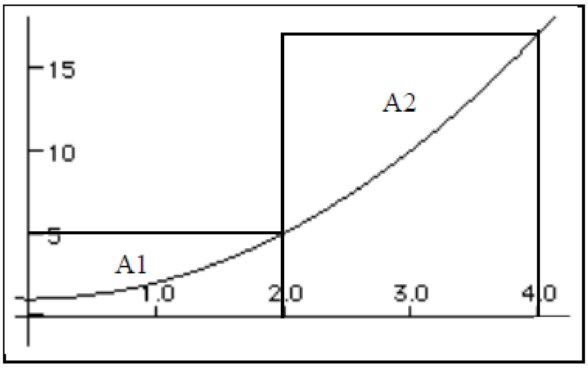
Area $\approx A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8$

Area $\approx b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4 + b_5h_5 + b_6h_6 + b_7h_7 + b_8h_8$

Area $\approx b_1 f(.5) + b_2 f(1) + b_3 f(1.5) + b_4 f(2) +$
$b_5 f(2.5) + b_6 f(3) + b_7 f(3.5) + b_8 f(4) =$

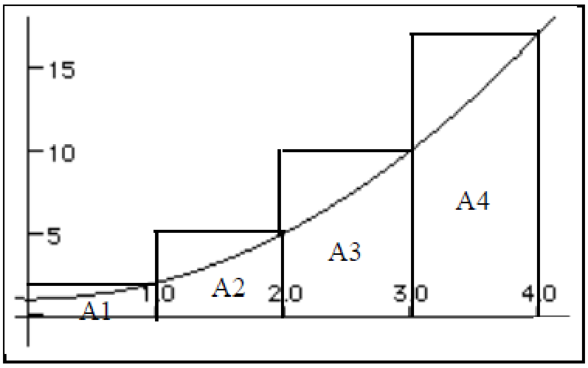


Estimate the area **30**



Area $\approx A1 + A2 = b_1h_1 + b_2h_2$

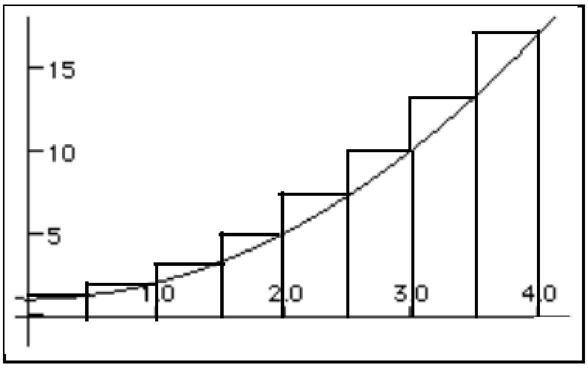
Area $\approx b_1 f(2) + b_2 f(4) = 44$



Area $\approx A1 + A2 + A3 + A4$

Area $\approx b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4$

Area $\approx b_1 f(1) + b_2 f(2) + b_3 f(3) + b_4 f(4) = 34$



Area $\approx A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8$

Area $\approx b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4 + b_5h_5 + b_6h_6 + b_7h_7 + b_8h_8$

Area $\approx b_1 f(.5) + b_2 f(1) + b_3 f(1.5) + b_4 f(2) +$

$b_5 f(2.5) + b_6 f(3) + b_7 f(3.5) + b_8 f(4) = 29.5$

As you go through this process, several things should be apparent:

- Drawing the function is not really necessary.
- The more rectangles you create, the more work you have to do. It is just a lot of arithmetic.
- In each case, the base is same allowing you to factor it out. For instance in the last case above,

$$b_1 f(.5)+b_2 f(1)+b_3 f(1.5)+b_4 f(2)+b_5 f(2.5)+b_6 f(3)+b_7 f(3.5)+b_8 f(4)=$$
$$b\left[f(.5)+f(1)+f(1.5)+f(2)+f(2.5)+f(3)+f(3.5)+f(4)\right]$$

- The more rectangles you create, the more accurate the area should be. So it should be apparent that

$$\text{True Area } = \lim_{n\to\infty}\sum_{i=1}^{n} A_i$$

Examples) Find the area under the following functions using the indicated number of rectangles:

1) $f(x)=3x+1$ on $[1,5]$

a) 4 b) 8

2. $f(x)=x^2+3$ on $[2,5]$

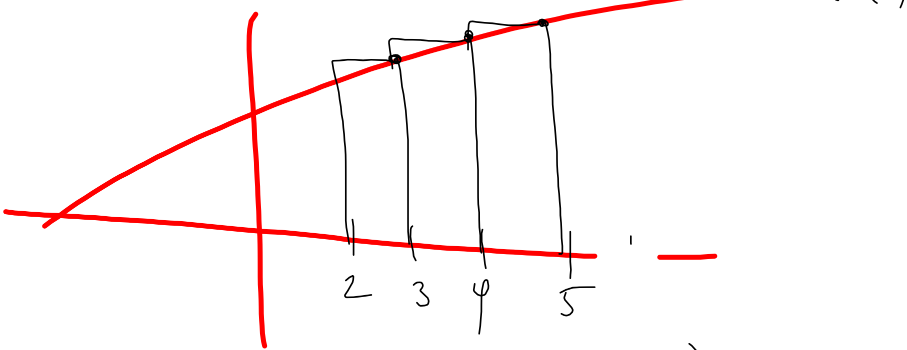
a) 3 ✓ b) 6

3. $f(x)=x^2-3x-2$ on $[4,6]$

a) 8

$f(x) = x^2 + 5 \quad [2,5]$

a) 3



$A_3 \approx 1\big(f(3) + f(4) + f(5)\big)$

$\approx 1(14 + 21 + 30)$

$= 65$

b)

As you go through this process, several things should be apparent:

- Drawing the function is not really necessary.
- The more rectangles you create, the more work you have to do. It is just a lot of arithmetic.
- In each case, the base is same allowing you to factor it out. For instance in the last case above,

$$b_1 f(.5) + b_2 f(1) + b_3 f(1.5) + b_4 f(2) + b_5 f(2.5) + b_6 f(3) + b_7 f(3.5) + b_8 f(4) =$$
$$b\left[f(.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4)\right]$$

- The more rectangles you create, the more accurate the area should be. So it should be apparent that

$$\text{True Area } = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

Examples) Find the area under the following functions using the indicated number of rectangles:

1) $f(x) = 3x+1$ on $[1,5]$ 2. $f(x) = x^2 + 3$ on $[2,5]$ 3. $f(x) = x^2 - 3x - 2$ on $[4,6]$

a) 4 b) 8 a) 3 b) 6 a) 8