

Calc WH  
Intro to U-Substitution

Name \_\_\_\_\_  
date \_\_\_\_\_ PD \_\_\_\_\_

Do now:

$$1. \frac{d}{dx} \left( \frac{x^4}{4} + 2x \right) = \frac{1}{4} \cdot 4x^3 + 2 \\ = x^3 + 2$$

$$2. \int (x^3 + 2) dx = \frac{x^4}{4} + 2x + C$$

$$3. \frac{d}{dx} \left[ \frac{1}{12} (x^2 - 1)^6 \right] =$$

$$= \frac{1}{12} \cdot 6(x^2 - 1)^5 \cdot 2x \\ = x(x^2 - 1)^5$$

$$4. \int x(x^2 - 1)^5 dx$$

$$= \frac{1}{12} (x^2 - 1)^6 + C$$

(16)

$$\int \frac{x^2 - 4x - 1}{2x^3} dx$$
$$= \int \left( \frac{x^2}{2x^3} - \frac{4x}{2x^3} - \frac{1}{2x^3} \right) dx$$
$$= \int \left( \frac{1}{2}x^0 - 2x^{-2} - \frac{1}{2}x^{-3} \right) dx$$
$$= \frac{1}{2}x + 2x^{-1} + \frac{1}{4}x^{-2} + C$$

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u-Substitution - Classwork

When you take derivatives of more complex expressions, you frequently have to use the chain rule to differentiate. The integration equivalent of the chain rule is called *u-substitution*. *u-substitution* allows you integrate expressions which do not appear integratable.

1)  $\int x(x^2 - 1)^5 dx$  Set up a  $u = \underline{x^2 - 1}$  Find  $\frac{du}{dx} = \underline{2x}$ . Solve for  $du = \underline{2x dx}$

You need to manufacture your  $du$  in the original expression. So you will have to multiply by  $\underline{2}$  on the inside and thus multiply by  $\underline{\frac{1}{2}}$  on the outside. Now change everything to  $u$ .

Now integrate in terms of  $u$ .

Finally, change back to the variable  $x$  and add  $C$ .

$$\frac{1}{2} \int \underline{2x} (x^2 - 1)^5 \underline{dx}$$

$$= \frac{1}{2} \int u^5 du$$

$$= \frac{1}{2} \cdot \frac{1}{6} u^6 + C$$

$$= \frac{1}{2} \cdot \frac{1}{6} (x^2 - 1)^6 + C$$

$$= \frac{1}{12} (x^2 - 1)^6 + C$$

$$2) \int (3x-2)^4 dx$$

$$u = 3x - 2$$

$$du = 3 dx$$

$$= \frac{1}{3} \int u^4 du$$

$$= \frac{1}{3} \frac{u^5}{5} + C$$

$$= \frac{1}{15} (3x-2)^5 + C$$

$$3) \int \sqrt{5x-2} dx$$

$$u = 5x - 2$$

$$du = \underline{\underline{5 dx}}$$

$$= \frac{1}{5} \int \sqrt{5x-2} dx$$

$$= \frac{1}{5} \int \sqrt{u} du$$

$$= \frac{1}{5} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{15} (5x-2)^{3/2} + C$$

4)  $\int 4(6x-1)^{2/3} dx$

$$u = 6x - 1$$

$$du = 6 dx$$

$$= \frac{4}{6} \int u^{2/3} du$$

$$= \frac{2}{3} \cdot \frac{3}{5} u^{5/3} + C$$

$$= \frac{2}{5} (6x-1)^{5/3} + C$$

5)  $\int x\sqrt{x^2-2} dx$

$$u = x^2 - 2$$

$$du = 2x dx$$

$$= \frac{1}{2} \int \sqrt{u} du$$

$$= \frac{1}{2} \int u^{1/2} du$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{3} (x^2-2)^{3/2} + C$$

$$6) \int x^2 \sqrt{1-4x^3} dx$$

$$u = 1 - 4x^3$$

$$du = -12x^2 dx$$

$$= -\frac{1}{12} \int u^{1/2} du$$

$$= -\frac{1}{12} \cdot \frac{2}{3} u^{3/2} + C$$

$$= -\frac{1}{18} \left( 1 - 4x^3 \right)^{3/2} + C$$

$$\int \frac{4x}{\sqrt{2x^2-1}} dx$$

$$u = 2x^2 - 1$$

$$du = 4x dx$$

$$= \frac{1}{4} \int \frac{1}{u^{1/2}} du$$

$$= \frac{1}{4} \int u^{-1/2} du$$

$$= \frac{1}{4} \cdot \frac{3}{2} u^{2/3} + C$$

$$= \frac{3}{8} \left( 2x^2 - 1 \right)^{2/3} + C$$

$$8) \int x^{1/2} (x^{3/2} + 2)^9 dx$$

$$u = x^{3/2} + 2$$

$$du = \frac{3}{2} x^{1/2} dx$$

$$= \frac{2}{3} \int u^9 du$$

$$= \frac{2}{3} \cdot \frac{1}{10} u^{10} + C$$

$$= \frac{1}{15} (x^{3/2} + 2)^{10} + C$$

$$9) \int (x+2) \sqrt{x^2 + 4x - 3} dx$$

$$u = x^2 + 4x - 3$$

$$du = (2x + 4) dx$$

$$= 2(x + 2) dx$$

$$= \frac{1}{2} \int u^{1/2} du$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{3} (x^2 + 4x - 3)^{3/2} + C$$

$$10) \int (x+2)\sqrt{x-4} \, dx$$

$$11) \int \frac{x-5}{\sqrt{x-6}} \, dx$$



$$12) \int \frac{x^2}{\sqrt{x+1}} dx$$

$$13) \int \cos 4x \, dx$$

$$14) \int 3\sin(1-3x) \, dx$$

$$15) \int \sin^3 x \cos x \, dx$$

$$16) \int \tan 10x \sec 10x \, dx$$

$$17) \int \tan^2 x \sec^2 x \, dx$$

$$\int \tan 10x \sec 10x \, dx \quad u = \tan 10x$$

$du = 10 \sec^2 10x$

~~$\frac{1}{10 \sec 10x}$~~   ~~$\frac{10 \sec 10x}{10 \sec 10x}$~~   $\int \tan 10x \sec 10x \, dx$





$$18) \int \sin x \sqrt{\cos x} \, dx$$

$$19) \int \frac{\cos x}{\sqrt{1 - \sin x}} \, dx$$

**u-Substitution - Homework**

1.  $\int \sqrt{x-2} \, dx$

2.  $\int (2x+3)^{11} \, dx$

3.  $\int \sqrt{5x-1} \, dx$

4.  $\int \sqrt[3]{6x+1} \, dx$

5.  $\int 5(3-4x)^{\frac{2}{3}} \, dx$

6.  $\int \frac{dx}{(8x-1)^3}$



$$\begin{aligned} \textcircled{1} \quad \int (x+5)^5 dx & \quad u = x+5 \\ & \quad du = dx \\ & = \int u^5 du = \frac{u^6}{6} + C = \frac{(x+5)^6}{6} + C \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \int 2x^5 dx & = 2 \int x^5 dx = 2 \cdot \frac{x^6}{6} + C \\ & = \frac{x^6}{3} + C \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \frac{1}{2} \int \sqrt{2x+5} dx & \quad u = 2x+5 \\ & \quad du = 2 dx \\ & = \frac{1}{2} \int \sqrt{u} du = \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C \\ & = \frac{1}{3} u^{3/2} + C \\ & = \frac{1}{3} (2x+5)^{3/2} + C \end{aligned}$$

$$7. \int x(x^2 + 2)^6 dx$$

$$8. \int 6x^2 \sqrt{3x^3 - 1} dx$$

$$9. \int \left(1 + \frac{1}{x}\right)^3 \left(\frac{1}{x^2}\right) dx$$

$u = 1 + \frac{1}{x}$   
 $du = -\frac{1}{x^2} dx$

$$- \int u^3 du$$

$$10. \int x^{1/3} \left(x^{4/3} + 9\right)^8 dx$$

$$11. \frac{2}{3} \int \sqrt{4 - \frac{3}{5}x} dx$$

$$12. \int (3x + 15) \sqrt{x^2 + 10x + 4} dx$$

13.  $\int (x+2)\sqrt{x-2} \, dx$

14.  $\int \frac{x^2}{\sqrt{x-4}} \, dx$

15.  $\int \sin 5x \, dx$

16.  $\int \cos \frac{x}{2} \, dx$

17.  $\int \frac{1}{3} \sec^2 8x \, dx$

18.  $\int \sin 4x \cos 4x \, dx$

19.  $\int \cos^3 x \sin x \, dx$

20.  $\int \tan x \sec^2 x \, dx$

21.  $\int \sqrt{\cos 6x} \sin 6x \, dx$

22.  $\int \frac{\sin x}{(4 - \cos x)^3} \, dx$

**u-Substitution - Homework**

1.  $\int \sqrt{x-2} \, dx$

$$\frac{2(x-2)^{\frac{3}{2}}}{3} + C$$

2.  $\int (2x+3)^{11} \, dx$

$$\frac{(2x+3)^{12}}{24} + C$$

3.  $\int \sqrt{5x-1} \, dx$

$$\frac{2(5x-1)^{\frac{3}{2}}}{15} + C$$

4.  $\int \sqrt[3]{6x+1} \, dx$

$$\frac{(6x+1)^{\frac{4}{3}}}{8} + C$$

5.  $\int 5(3-4x)^{\frac{2}{3}} \, dx$

$$\frac{-3(3-4x)^{\frac{5}{3}}}{4} + C$$

6.  $\int \frac{dx}{(8x-1)^3}$

$$\frac{-1}{16(8x-1)^2} + C$$

$$7. \int x(x^2 + 2)^6 dx$$

$$\frac{(x^2 + 2)^7}{14} + C$$

$$9. \int \left(1 + \frac{1}{x}\right)^3 \left(\frac{1}{x^2}\right) dx$$

$$\frac{-\left(1 + \frac{1}{x}\right)^4}{4} + C$$

$$11. \frac{2}{3} \int \sqrt{4 - \frac{3}{5}x} dx$$

$$\frac{-20\left(4 - \frac{3}{5}x\right)^{\frac{3}{2}}}{27} + C$$

$$8. \int 6x^2 \sqrt{3x^3 - 1} dx$$

$$\frac{4(3x^3 - 1)^{\frac{3}{2}}}{9} + C$$

$$10. \int x^{\frac{1}{3}} \left(x^{\frac{4}{3}} + 9\right)^8 dx$$

$$\frac{\left(x^{\frac{4}{3}} + 9\right)^9}{12} + C$$

$$12. \int (3x + 15) \sqrt{x^2 + 10x + 4} dx$$

$$\left(x^2 + 10x + 4\right)^{\frac{3}{2}} + C$$

$$13. \int (x+2)\sqrt{x-2} \, dx$$

$$\frac{2(x-2)^{\frac{5}{2}}}{5} + \frac{8(x-2)^{\frac{3}{2}}}{3} + C$$

$$14. \int \frac{x^2}{\sqrt{x-4}} \, dx$$

$$\frac{2(x-4)^{\frac{5}{2}}}{5} + \frac{16(x-4)^{\frac{3}{2}}}{3} + 32(x-4)^{\frac{1}{2}} + C$$

$$15. \int \sin 5x \, dx$$

$$\frac{-\cos 5x}{5} + C$$

$$16. \int \cos \frac{x}{2} \, dx$$

$$2 \sin \frac{x}{2} + C$$

$$17. \int \frac{1}{3} \sec^2 8x \, dx$$

$$\frac{\tan 8x}{24} + C$$

$$18. \int \sin 4x \cos 4x \, dx$$

$$\frac{\sin^2 4x}{8} + C \text{ or } \frac{-\cos^2 4x}{8} + C$$

$$19. \int \cos^3 x \sin x \, dx$$

$$\frac{-\cos^4 x}{4} + C$$

$$20. \int \tan x \sec^2 x \, dx$$

$$\frac{\tan^2 x}{2} + C$$

$$21. \int \sqrt{\cos 6x} \sin 6x \, dx$$

$$\frac{-(\cos(6x))^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$22. \int \frac{\sin x}{(4 - \cos x)^3} \, dx$$

$$\frac{-1}{2(4 - \cos x)^2} + C$$