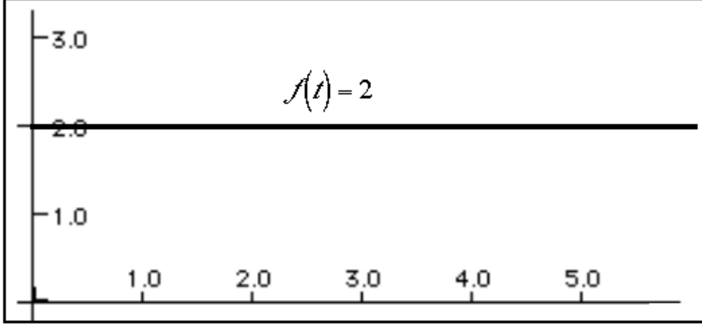
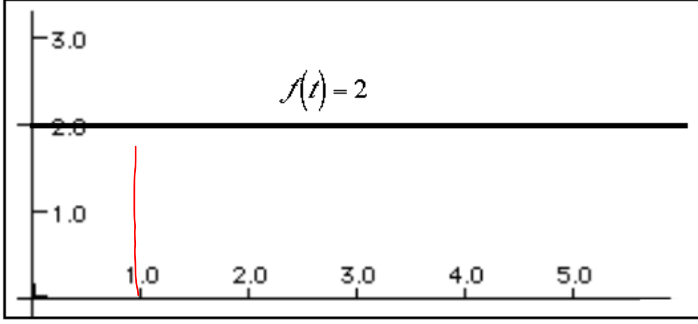
The Accumulation Function - Classwork

Now that we understand that the concept of a definite integral is nothing more than an area, let us consider a very special type of area problem. Suppose we are given a function $f(x)=2$. We know that to be a horizontal line at $y = 2$. First, realize that the equation of the graph of $f(x)=2$ is the same as $f(t)=2$ is the same as $f(k)=2$. Whether we use x, t, or k, it does not matter. The graph is still a horizontal line. We are used to using x but we will see a good reason that we will occasionally be using another letter.



Consider the expression

$$\int_0^x f(t)\,dt$$



Consider the expression

$$\int_0^x f(t)\,dt$$

Above, we have the graph $f(t) = 2$. We now want to consider the expression $\int_0^x f(t)\,dt$. What is this? It is a function of x. As x changes, $\int_0^x f(t)\,dt$ changes as well. Complete the chart below.

x	0	1	2	3	4	5
$\int_0^x f(t)\,dt$	0	2	4	6	8	10

It should be apparent that as x gets bigger, $\int_0^x f(t)\,dt$ increases as well. What we appear to be doing is "accumulating area" and we call $\int_0^x f(t)\,dt$ the accumulation function. Finally, it should be obvious why we use $f(t) = 2$ to describe our line rather than $f(x) = 2$. $\int_0^x f(x)\,dx$ would be confusing.

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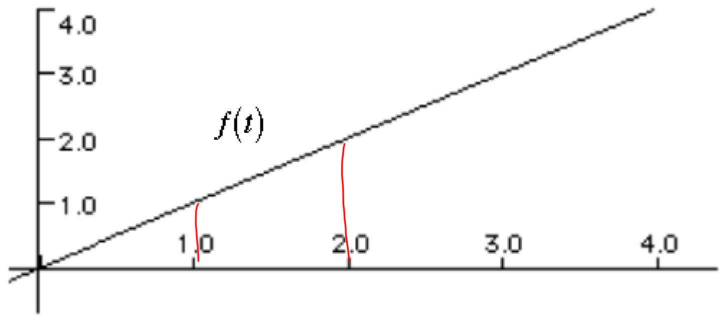
Let's calculate the value of

$\int_0^x f(t)\,dt$ for $x = 0$ to 4

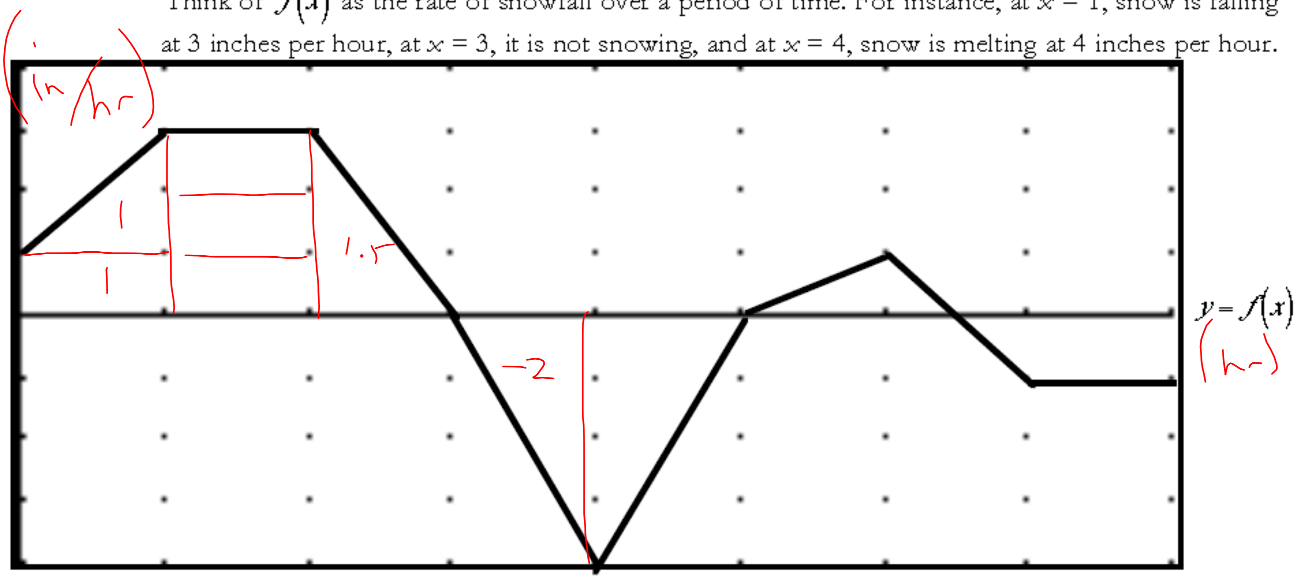
Complete the chart below



x	0	1	2	3	4
$\int_0^x f(t)\,dt$	0	$\frac{1}{2}$	2	$\frac{9}{2}$	8

Example 1) Let $\boxed{F(x)=\int_0^x f(t)\,dt}$ where the graph of $f(x)$ is below. Remember $f(x)$ is the same thing as $f(t)$.

Think of $f(x)$ as the rate of snowfall over a period of time. For instance, at x = 1, snow is falling at 3 inches per hour, at x = 3, it is not snowing, and at x = 4, snow is melting at 4 inches per hour.



a. Complete the chart. In the snow analogy, $F(x)$ represents the accumulation of snow over time.

x	0	1	2	3	4	5	6	6.5	7	8
$F(x)$	0	2	5	6.5	4.5					