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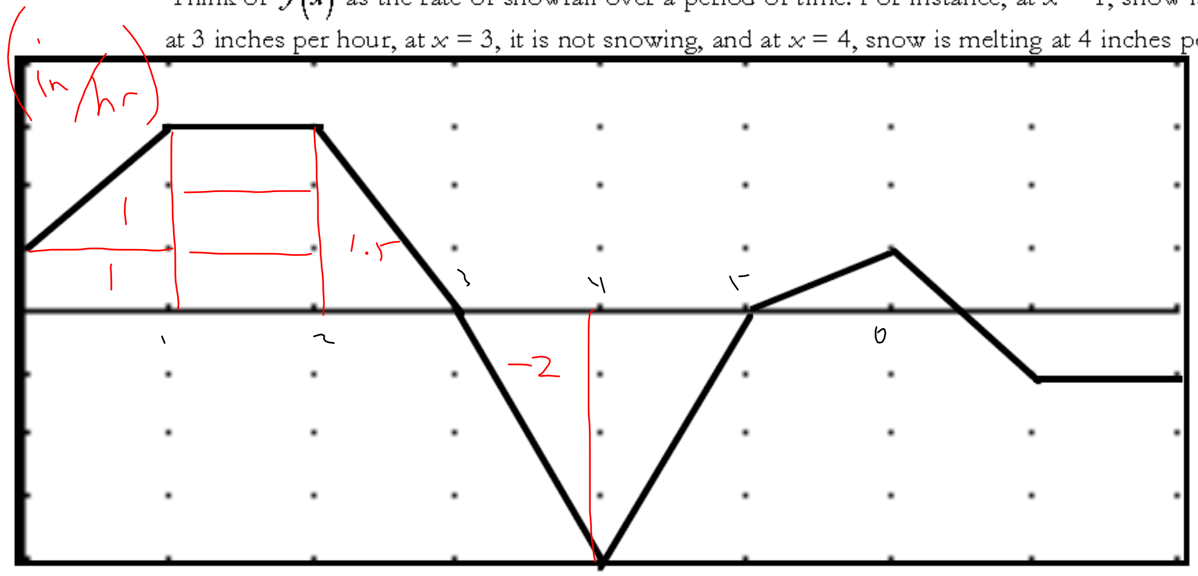
Example 1) Let $\boxed{F(x) = \int_0^x f(t)\,dt}$ where the graph of $f(x)$ is below. Remember $f(x)$ is the same thing as $f(t)$.

Think of $f(x)$ as the rate of snowfall over a period of time. For instance, at x = 1, snow is falling at 3 inches per hour, at x = 3, it is not snowing, and at x = 4, snow is melting at 4 inches per hour.



F'

$y = f(x)$

(hr)

a. Complete the chart. In the snow analogy, $F(x)$ represents the accumulation of snow over time.

x	0	1	2	3	4	5	6	6.5	7	8
$F(x)$	0	2	5	6.5	4.5	2.5	3	3.25	3	2

b. Now let's consider $F(x) = \frac{d}{dx}\int_0^x f(t)\,dt$ $= f(x)$. If we take the derivative of an integral, what would you expect to happen? cancel out So $F(x) = \frac{d}{dx}\int_0^x f(t)\,dt$ is the same thing as $f(x)$.

Knowing that, let's complete the chart.

x	0	1	2	3	4	5	6	6.5	7	8
$f(x) = F(x)$	1	3	3	0	−4	0	1	0	−1	−1

c. On what subintervals of [0, 8] is F increasing? Decreasing?

rel max rel min rel max

inc: (0, 3), (5, 6.5)

F′: + 0 − 0 + 0 − on 0, 3, 5, 6.5, 8

dec: (3, 5), (6.5, 8)

d. Where in the interval [0, 8] does $\boxed{F}$ achieve its minimum and maximum value? What are those values?

F(3) = 6.5 max

F(6.5) = 3.25

F(0) = 0 min

F(5) = 2.5

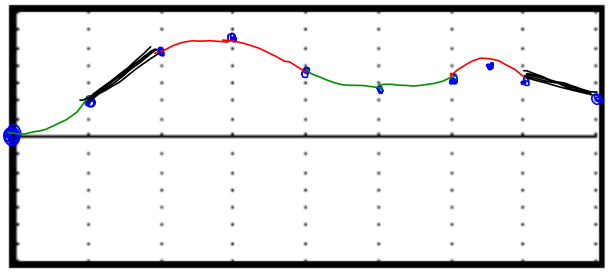
F(8) = 2

f. Find the concavity of F and any inflection points. Justify your answer

F″: CU CD IP CU IP CD

+ 0 − + − 0 on 0, 1, 2, 4, 6, 7, 8

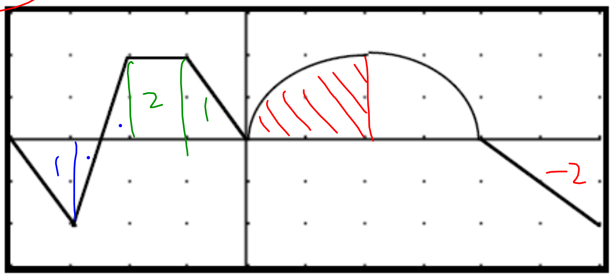
h. Sketch a rough graph of $F(x)$



Example 2) Let $F(x) = \int_0^x f(t)\,dt$ where f is the function graphed below (consisting of lines and a semi-circle)

$F' = f$

$F'(4) = f(4)$



$y = f(x)$

Find the following:

a) $F(0) = 0$ b) $F(2) = \pi$ c) $F(4) = 2\pi$ d) $F(6) = 2\pi - 2$

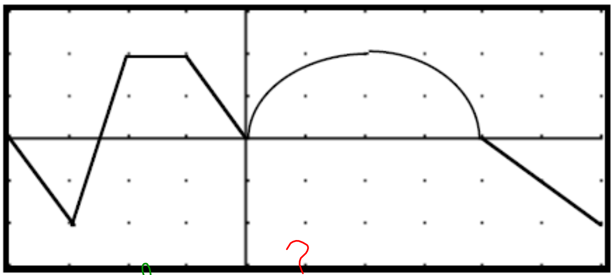
e) $F(-1) = -1$ f) $F(-2) = -3$ g) $F(-3) = -3$ h) $F(-4) = -2$

i) $F'(4) = 0$ j) $F'(2) = 2$ k) $F'(6) = -2$ l) $F'(-3) = -2$

Example 2) Let $F(x)=\int_0^x f(t)\,dt$ where f is the function graphed below (consisting of lines and a semi-

$F'' = f'$

$F' = f(x)$



$y=f(x)$

?

rel min

rel max

?

m) F'

-4		-2.5		0		4		6
0	−	0	+	0	+	0	−	

gnochhi
gnocchi

F is inc on $(-2.5, 0), (0, 4)$ bc. that's where $F' > 0$.

F is dec on $(-4, -2.5), (4, 6)$ bc. that's where F' is neg.

n) $F(-2.5) = -3.5$ F is a minimum at $x = -2.5$.
The min value is -3.5.

$F(6) = 2\pi - 2$

o) F is a max at $x=4$. The max value is 2π.

$F(-4) = -2$

$F(4) = 2\pi$

p) F''

-4		-3		-2		-1		0		2		6
	−		+	0	−				+		−	

F is CU on $(-3, -2), (0, 2)$
bc. that's where: $F'' > 0$

F' is inc

~~f is inc~~

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