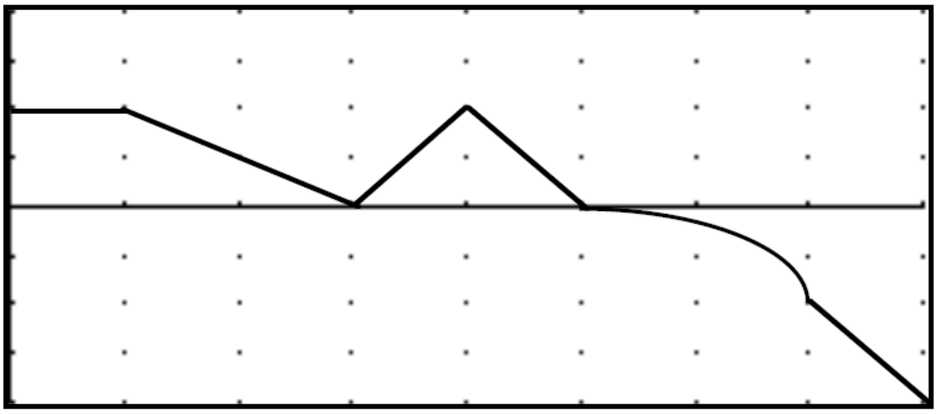
The Accumulation Function - Homework



$y = f(x)$

1. Let $F(x) = \int_0^x f(t)\,dt$ where the graph of $f(x)$ is above (the graph consists of lines and a quarter circle)

a. Complete the chart

x	0	1	2	3	4	5	7	8
$F(x)$	0	2	3.5	4	5	6	2+π	π-1
$F'(x)$	**2**	**2**	**1**	**0**	**2**	**0**	**-2**	**-4**

b. On what subintervals of [0, 8] is F increasing? Decreasing? Justify your answer.

Increasing where $F' > 0$: $\boxed{(0,3)\cup(3,5)}$ Decreasing where $F' < 0$: $\boxed{(5,8)}$

c. Where in the interval [0, 8] does F achieve its minimum value? What is the minimum value? Justify answer.

$\boxed{0 \text{ at } x = 0}$

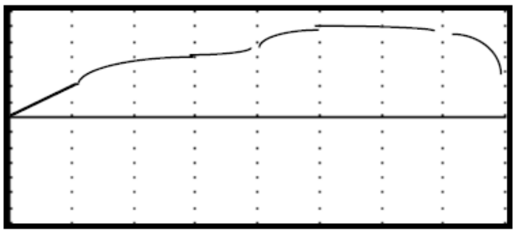
d. Where in the interval [0, 8] does F achieve its maximum value? What is the maximum value? Justify answer.

$\boxed{6 \text{ at } x = 5}$

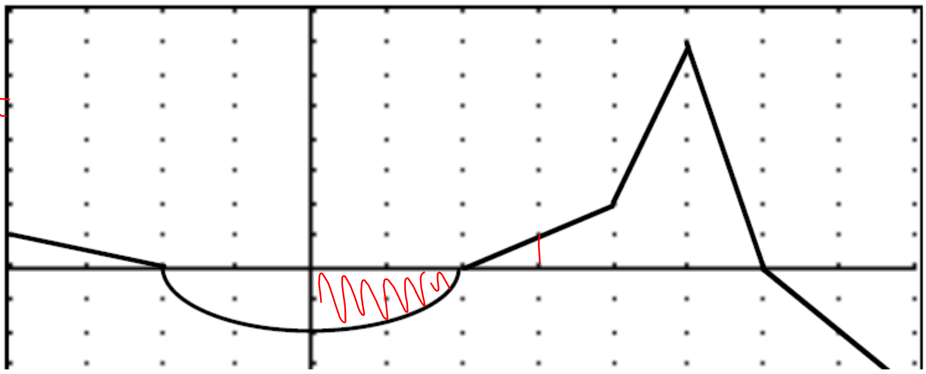
e. Find the concavity of F and any inflection points. Justify answers.

Up where F' increasing: $\boxed{(3,4)}$ Down where F' dcreasing: $\boxed{(1,3)\cup(4,8)}$

f. Sketch a rough graph of $F(x)$



$\int_0^x f(t)\,dt$



2. Let $F(x)=\int_0^x f(t)\,dt$ where the graph of $f(x)$ is above (the graph consists of lines and a semi-circle)

a. Complete the chart

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
$F(x)$	π-1	π-.25	π	1.8	0	-1.8	-π	-π+.5	2-π	6.5-π	10-π	9-π	6-π
$F'(x)$	**1**	**.5**	**0**	**-1.8**	**-2**	**-1.8**	**0**	**1**	**2**	**7**	**0**	**-2**	**-4**

b. On what subintervals of $[-4,8]$ is F increasing? Decreasing?

Increasing where $F'>0$: $\boxed{(-4,-2)\cup(2,6)}$ Decreasing where $F'<0$: $\boxed{(-2,2)\cup(6,8)}$

c. Where in the interval $[-4,8]$ does F achieve its minimum value? What is the minimum value? Justify answer.

Relative minimum at $x=2$ as $F'<0$ if $x<2$, $F'>0$ if $x>2$
$F(2)=-\pi$, $F(-4)=\pi-1$, $F(8)=6-\pi$. Abs. Min $=-\pi$ at $x=2$.

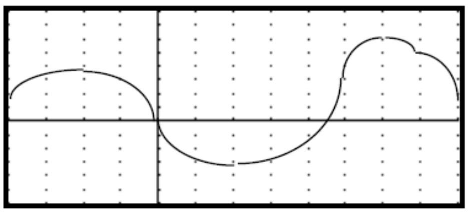
d. Where in the interval $[-4,8]$ does F achieve its maximum value? What is the maximum value? Justify answer.

Relative maximum at $x=-2$ as $F'>0$ if $x<-2$, $F'<0$ if $x>-2$
Relative maximum at $x=6$ as $F'>0$ if $x<6$, $F'<0$ if $x>6$
$F(-2)=\pi$, $F(6)=10-\pi$, $F(-4)=\pi-1$, $F(8)=6-\pi$. Abs. Max $=10-\pi$ at $x=6$.

e. On what subintervals of does F concave up and concave down? Find its inflection points. Justify answers.

Concave up when F' increasing : $(0,5)$
Concave down when F' decreasing : $(-4,0)\cup(5,8)$

f. Sketch a rough graph of $F(x)$



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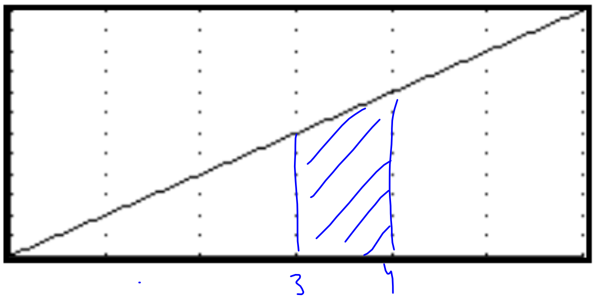
The Fundamental Theorem of Calculus - Classwork

Example 1) On your calculator, graph the function $y = 2x$ on the interval [0, 6]. You should get the graph below.



Now let

$$F(x) = \int_0^x 2t\,dt$$

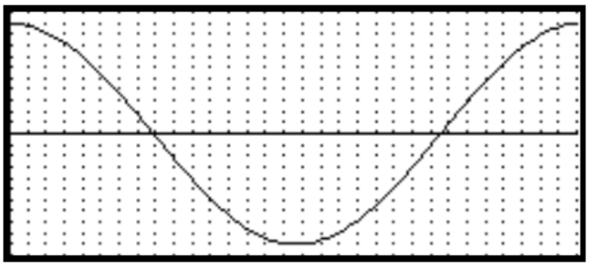
This is the accumulated area under the graph of the function to the left. Complete the chart below.

x	0	1	2	3	4	5	6
$F(x)$	0	1	4	9	16	25	36

What is the relationship that you see between x and $F(x)$? $= x^2$

So it appears that $\int_0^x 2t\,dt = x^2$

Example 2) On your calculator, graph $y = \cos x$ on the interval [0, 2π]. You should get the graph below.



Now let

$$F(x) = \int_0^x \cos t \, dt$$

This is the accumulated area under the curve $y = \cos x$

On your calculator, set your accuracy to 2 decimal places and define Y2 as FnInt(Y1,X,0,X). FnInt is located in $\boxed{\text{Math}}$ 9. Then go to $\boxed{\text{2nd}}$ $\boxed{\text{TBLSET}}$, define TblStart = 0 and ΔTbl = .2. Then press $\boxed{\text{2nd}}$ $\boxed{\text{Table}}$. Your accumulated area should be located in Y2.

b. We need to get this set of values in to a list. Press $\boxed{\text{STAT}}$ $\boxed{\text{EDIT}}$ L1 can be defined as $\boxed{\text{2nd}}$ $\boxed{\text{List}}$ Seq(X,X,0,6.2,.2). L2 is defined as Y2(L1). It will take a few seconds to compute these values. Now go to $\boxed{\text{Stat Plot}}$, turn Plot1 on, and perform a scatterplot with L1 vs. L2

c. Go to $\boxed{\text{Y=}}$ and turn both Y1 and Y2 off. Now graph the function.

d. Guess the function whose scatterplot you have _______________ So it appears that $\int_0^x \cos t \, dt =$ ____________

We are now ready to make the connection to area under a curve and indefinite integration introduced earlier. This is accomplished through the Fundamental Theorem of Calculus (Newton and Leibniz).

The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a,b]$, and F is an antiderivative of f on $[a,b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

What the Fundamental Theorem of Calculus (FTC) allows you to do is to find the area under the curve by the process of integration and not having to use the limit of a sum.

Examples) Find the value of the definite integrals below.

1) $\int_1^2 x^2\,dx = \left.\frac{x^3}{3}\right|_1^2$

$= \frac{8}{3} - \frac{1}{3}$

$= 7/3$

2) $\int_{-1}^{3} (3x^2 - 2x - 1)\,dx$

$= \left.(x^3 - x^2 - x)\right|_{-1}^{3}$

$= (27 - 9 - 3) - (-1 - 1 + 1)$

$= 16$

3) $\int_0^9 \sqrt{x}\,dx = \left.\frac{2}{3}x^{3/2}\right|_0^9$

$= \frac{2}{3}(27 - 0)$

$= 18$

4) $\int_{5}^{10} \frac{2}{3x^2}\,dx =$

$= \frac{2}{3}\int x^{-2}\,dx$

$= \frac{2}{3}\ \frac{x^{-1}}{-1}\Big|_{5}^{10}$

$= -\frac{2}{3}\left(\frac{1}{10} - \frac{1}{5}\right)$

$= -\frac{2}{3}\left(\frac{-1}{10}\right)$

$= 1/15$

5) $\int_{-1}^{1} (2x-1)^2\,dx$

$= \int_{-1}^{1} \left(4x^2 - 4x + 1\right)dx$

$= \left(\frac{4}{3}x^3 - 2x^2 + x\right)\Big|_{-1}^{1}$

$= \left(\frac{4}{3} - 2 + 1\right) - \left(-\frac{4}{3} - 2 - 1\right)$

$= \frac{1}{3} + \frac{13}{3}$

$= 14/3$

6) $\int_{1}^{8} \frac{x-2}{\sqrt[3]{x}}\,dx = \int_{1}^{8}\left(x^{2/3} - 2x^{-1/3}\right)dx$

$= \left(\frac{3}{5}x^{5/3} - 3x^{2/3}\right)\Big|_{1}^{8}$

$= \left(\frac{96}{5} - 12\right) - \left(\frac{3}{5} - 3\right)$

$= \frac{36}{5} + 12/5$

$= 48/5$

7) $\int_0^{\pi} (2+\sin x)\, dx$

$= \left(2x - \cos x\right)\Big|_0^{\pi}$

$= \left(2\pi - -1\right) - \left(0 - 1\right)$

$= 2\pi + 1 + 1$

$= 2\pi + 2$

8) $\int_0^{\pi/4} (4x+\sec^2 x)\, dx$

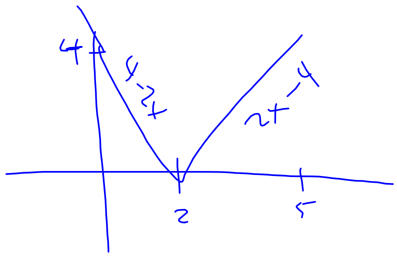
$= \left(2x^2 + \tan x\right)\Big|_0^{\frac{\pi}{4}}$

$= \left(\frac{\pi^2}{8} + 1\right) - \left(0 + 0\right)$

$= \frac{\pi^2}{8} + 1$

9) $\int_0^{5} |4-2x|\, dx$

$$\int_0^5 |4-2x|\,dx = \int_0^2 (4-2x)\,dx + \int_2^5 -(4-2x)\,dx$$



$$y = |4-2x|$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$= \begin{cases} 4-2x, & 4-2x > 0 \quad 2 > x \\ -(4-2x), & 4-2x < 0 \rightarrow 2 < x \end{cases}$$

$$= \begin{cases} 4-2x, & \underline{x < 2} \\ -(4-2x), & \underline{x \geq 2} \end{cases}$$

Technology: Your TI-84 calculators allow you to find the value of a definite integral. The command is $\boxed{\text{FnInt}}$ and is located as #9 in the $\boxed{\text{Math}}$ menu. The syntax of the statement is FnInt(expression in X, X, lower, upper). For instance, example # 1 above -- $\int_1^2 x^2\,dx$ would be expressed to the calculator as FnInt(X^2, X,1,2) yielding 2.333. You can also put your expression in Y1 and your statement would be FnInt(Y1, X,1,2). The calculator is finding the integral but not by the Fundamental Theorem of Calculus. It is merely performing the summation of rectangles or trapezoids many many times.

The Fundamental Theorem of Calculus - Homework

Find the value of the definite integrals below. Confirm using your calculator.

1. $\int_{0}^{1} 3x \, dx$
2. $\int_{-2}^{3} (x-5) \, dx$
3. $\int_{-1}^{4} (x^2+2x-1) \, dx$
4. $\int_{0}^{2} (2x-5)^2 \, dx$
5. $\int_{2}^{3} \left(\frac{4}{x^2}+1\right) dx$
6. $\int_{-2}^{-1} \left(x-\frac{1}{x^2}\right) dx$

7. $\int_{1}^{9} \frac{x-2}{\sqrt{x}}\,dx$

8. $\int_{-2}^{2} \sqrt[3]{x}\,dx$

9. $\int_{0}^{1} \left(t^{2/3} - t^{1/3}\right) dt$

10. $\int_{0}^{3} |x - 2|\,dx$

11. $\int_{-\pi/2}^{\pi/2} \cos x\,dx$

12. $\int_{0}^{\pi} \left(2x - \sin x\right) dx$

13. $\int_0^{\pi/2} \left(3\sin x - 2\cos x\right)\, dx$

14. $\int_0^{\pi/4} \left(x - \sec^2 x\right)\, dx$

15. $\int_0^{\pi/3} \sec\theta \tan\theta \, d\theta$