

HW Page 158

① -4

② $24/5$

③ $3/4$

④ $\frac{\pi}{12} - \frac{1}{4}$

⑤ $7/3$

⑥ 9

⑦ $17/2$

⑧ 3

⑨ $5/12$

⑩ 2

⑪ $1/4$

⑫ $1/2$

$$(13) \quad 2/3$$

$$(14) \quad 3/8$$

$$(15) \quad \frac{(ax^2+b)^{3/2} - b^{3/2}}{3a}$$

$$(16) \quad 2$$

$$(17) \quad 64/3$$

$$(18) \quad ?$$

$$(19) \quad 11/3$$

$$(20) \quad 22/3$$

$$(21) \quad -11/3$$

$$(22) \quad 11$$

$$(23) \quad 5$$

$$(24) \quad -11/3$$

$$(25) \quad 0$$

$$(26) \quad -11/3$$

$$(27) \quad -11$$

$$(28) \quad 0$$

$$(29) \quad 5$$

$$(30) \quad 0$$

Definite Integration with u-Substitution - Homework

Find the values of the following definite integrals. Verify using your calculator. Some will use u -substitution, others will not.

$$1. \int_{-2}^2 (x^3 - 1) \, dx$$

$$2. \int_0^4 x(\sqrt{x} - 1) \, dx$$

$$3. \int_0^{\pi/3} \sin(2x) \, dx$$

$$4. \int_0^{\pi/12} (1 - \cos 2x) \, dx$$

$$5. \int_0^1 2x(x^2 + 1)^2 \, dx$$

$$6. \int_0^3 x\sqrt{9-x^2} \, dx$$

$$\begin{aligned}
 & u = 9 - x^2 \\
 & du = -2x \, dx \\
 & = -\frac{1}{2} \int_{u=9}^0 u^{1/2} \, du \\
 & = -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} \Big|_9^0 \\
 & = -\frac{1}{3} \cdot (0 - 27) \\
 & = 9
 \end{aligned}$$

$$7. \int_0^5 |x-4| dx$$

$$8. \int_0^4 |x - \sqrt{x}| dx$$

$$9. \int_2^3 \frac{x}{(x^2-3)^2} dx$$

$$10. \int_0^4 \frac{dt}{\sqrt{2t+1}}$$

$$11. \int_0^{\pi/2} \cos^3 t \sin t dt$$

$$12. \int_0^{\sqrt{\pi/2}} t \sin(\pi - t^2) dt$$

$$\textcircled{7} \quad \int_0^5 |x-4| dx$$

$$= \int_0^4 (x-4) dx + \int_4^5 (x-4) dx$$

$$= \left(\frac{x^2}{2} - 4x \right) \Big|_0^4 + \left(\frac{x^2}{2} - 4x \right) \Big|_4^5$$

$$= - \left((8-16) - (0) \right) + \left(\left(\frac{25}{2} - 20 \right) - (8-16) \right)$$

$$= -(-8) + \left(-\frac{15}{2} + 8 \right)$$

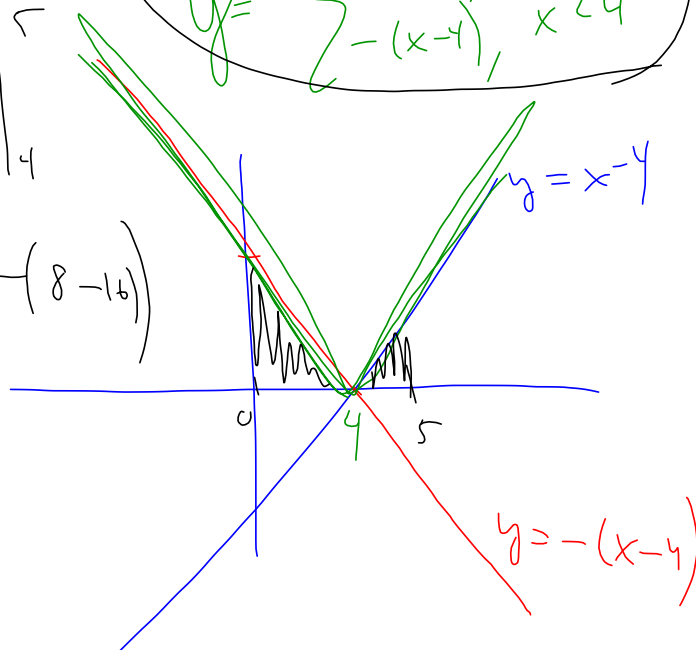
$$= 16 - \frac{15}{2}$$

$$= \textcircled{17/2}$$

$$y = |x-4|$$

$$y = \begin{cases} x-4, & x-4 > 0 \\ -(x-4), & x-4 < 0 \end{cases}$$

$$y = \begin{cases} x-4, & x > 4 \\ -(x-4), & x < 4 \end{cases}$$



$$\textcircled{8} \quad \int_0^4 |x - \sqrt{x}| dx$$

$$y = |x - \sqrt{x}|$$

$$y = \begin{cases} x - \sqrt{x}, & x - \sqrt{x} > 0 \\ -(x - \sqrt{x}), & x - \sqrt{x} < 0 \end{cases}$$

$$= -\int_0^1 (x - \sqrt{x}) dx + \int_1^4 (x - \sqrt{x}) dx$$

$$x - \sqrt{x} = 0$$

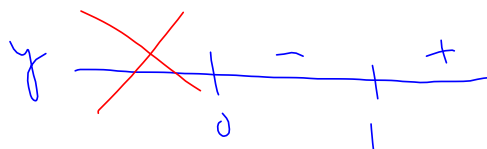
$$x = \sqrt{x}$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0 \quad x = 1$$



$$y = \begin{cases} x - \sqrt{x}, & x > 1 \\ -(x - \sqrt{x}), & 0 < x < 1 \end{cases}$$

$$13. \int_0^{\sqrt{\pi/2}} t \sin(\pi - t^2) dt$$

$$14. \int_0^{\pi/3} \cos x \sqrt{1 - \cos^2 x} dx$$

$$15. \int_0^1 x \sqrt{ax^2 + b} dx$$

$$u = ax^2 + b$$

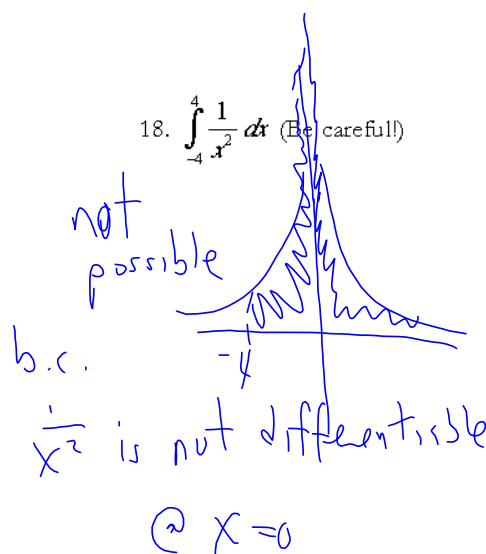
$$du = 2ax dx$$

$$= \frac{1}{2a} \int u^{1/2} du$$

$$16. \int_{\pi^2/4}^{\pi^2} \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

$$17. \int_0^4 |9 - x^2| dx$$

$$18. \int_{-4}^4 \frac{1}{x^2} dx \text{ (Be careful!)}$$



$$\int_0^4 |9 - x^2| dx$$

$$= \int_0^3 (9 - x^2) dx - \int_3^4 (9 - x^2) dx$$

$$y = |9 - x^2|$$

$$y = \begin{cases} 9 - x^2, & -3 < x < 3 \\ -(9 - x^2), & x < -3 \text{ or } x > 3 \end{cases}$$

$9 - x^2 > 0$

$9 - x^2 < 0$

$$y = 9 - x^2 = 0$$

$$(3 - x)(3 + x) = 0$$

$$\begin{array}{c} - \quad + \quad - \\ \hline -3 \quad \quad 3 \end{array}$$

If $\int_0^2 f(x) dx = \frac{11}{3}$ and $\int_0^6 f(x) dx = 15$, $f(x)$ is an even function (symmetric to the y -axis), find the following:

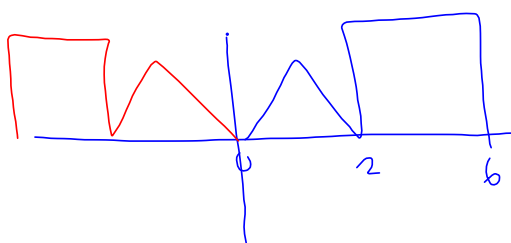
19. $\int_{-2}^0 f(x) dx$

20. $\int_{-2}^2 f(x) dx$

21. $\int_0^2 -f(x) dx$

22. $\int_{-2}^0 3f(x) dx$

23. $\int_0^6 f(3x) dx$



$= 3 \left(\frac{11}{3} \right) = 11$

$$\begin{aligned} \int_0^2 f(3x) dx & \quad u = 3x \\ & \quad du = 3 dx \\ & = \frac{1}{3} \int_0^6 f(u) du \\ & = \frac{1}{3} (15) \\ & = 5 \end{aligned}$$

If $\int_0^2 f(x) dx = \frac{11}{3}$ and $\int_0^6 f(x) dx = 15$, $f(x)$ is an odd function (symmetric to the origin), find the following:

24. $\int_{-2}^0 f(x) dx$ 25. $\int_{-2}^2 f(x) dx$ 26. $\int_0^2 -f(x) dx$ 27. $\int_{-2}^0 3f(x) dx$ 28. $\int_0^2 f(3x) dx$