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Straight Line Motion - Revisited - Classwork

When we looked at straight line motion in our derivative section, we generated a relationship between position, velocity, and acceleration. Given position function $s(t)$, the velocity function $v(t) = s'(t)$ and the acceleration function $a(t) = v'(t) = s''(t)$. We can now move in the opposite direction as well. If you have an object traveling in a straight line, its velocity can be written as $v(t) = \int a(t)\,dt + C$.

Examples) Find $v(t)$ given $a(t)$ and $v(0)$

1) $a(t) = 4t - 6$ and $v(0) = 3$

$v(t) = \int (4t-6)\,dt$

$v(t) = 2t^2 - 6t + C$

$v(0) = C = 3$

$\boxed{v(t) = 2t^2 - 6t + 3}$

2) $a(t) = \sin t + 2t$ and $v(0) = 5$

$v(t) = \int (\sin t + 2t)\,dt$

$v(t) = -\cos t + t^2 + C$

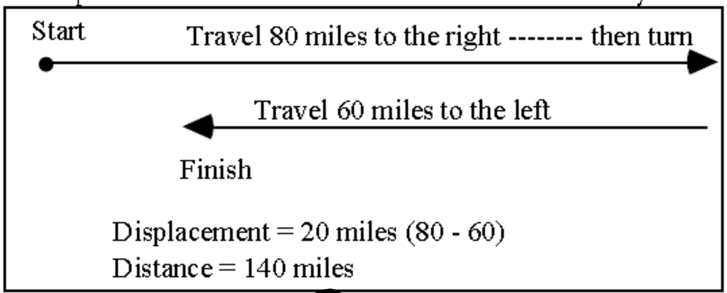
$v(0) = -1 + 0 + C = 5$

$C = 6$

$v(t) = -\cos t + t^2 + 6$

We now define two terms: distance: how far you travel between time $t = a$ and time $t = b$.

displacement: the difference in distance from where you start and where you stop.



Can displacement equal distance? How? yes if vel is always pos.

So if we are given a velocity function which is always positive, we know that distance = displacement. But if the velocity function is not always positive, we must find when the time interval when the velocity is negative (moving backwards) so we can find the distance traveled in that interval and subtract it (which will be subtracting a negative number or adding a positive one). This can be summarized as:

Given a particle moving on a straight line with velocity $v(t)$ between time $t = a$ and time $t = b$. Then

$$\text{Displacement} = \int_a^b v(t)\,dt \qquad \text{Distance} = \int_a^b \left| v(t) \right| dt$$

Given a particle moving on a straight line with velocity $v(t)$ between time $t = a$ and time $t = b$. Then

Displacement = $\int_a^b v(t)\,dt$ Distance = $\int_a^b \left| v(t) \right| dt$

Example 3) A particle is moving along a straight line with velocity $v(t) = t^2 - 7t + 10$ (ft/sec) Find the displacement and distance traveled on the time interval $[1, 7]$

Displacement (easy) Distance (more difficult)

$$\text{disp} = \int_1^7 (t^2 - 7t + 10)\,dt$$

$$= \left(\frac{t^3}{3} - \frac{7t^2}{2} + 10t\right)\Bigg|_1^7$$

$$= \left(\frac{343}{3} - \frac{343}{2} + 70\right) - \left(\frac{1}{3} - \frac{7}{2} + 10\right)$$

$$= 6 \text{ ft}$$

$$\text{dist} = \int_1^7 \left| t^2 - 7t + 10 \right| dt$$

$$\text{dist} = \int_1^7 \left| t^2 - 7t + 10 \right| dt$$

$$t^2 - 7t + 10 = 0$$

$$(t-2)(t-5) = 0$$

$$t = 2 \quad t = 5$$

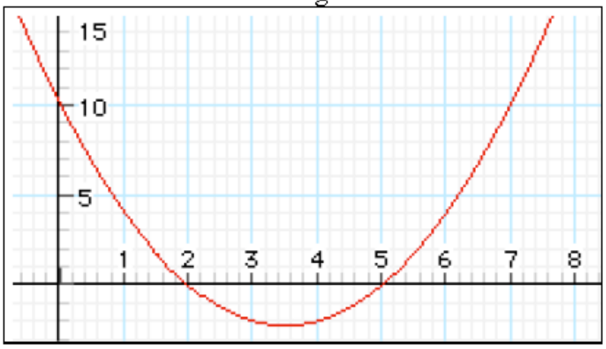
f: $+$ on the left of 2, $-$ between 2 and 5, $+$ on the right of 5

$$= \int_1^2 (t^2 - 7t + 10)\,dt$$

$$- \int_2^5 (t^2 - 7t + 10)\,dt$$

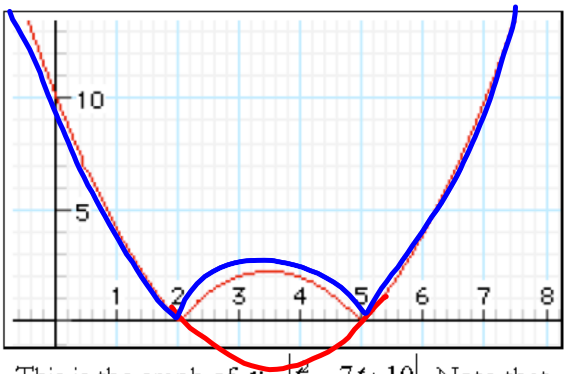
$$+ \int_5^7 (t^2 - 7t + 10)\,dt$$

This can be confirmed using the calculator.



This is the graph of $v = t^2 - 7t + 10$, notice that v is negative (going backward) between $t = 2$ and $t = 5$. When you take the integral $\int_1^7 \left(t^2 - 7t + 10\right) dt$, this will be taken into account.

If you are allowed to use your calculator, you use FnInt(X^2-7X+10,X,1,7)



This is the graph of $v = \left|t^2 - 7t + 10\right|$. Note that between $t = 2$ and $t = 5$, the function is now above the axis and distance is positive. So the integral is

$$\int_1^7 \left|t^2 - 7t + 10\right| dt =$$

$$\int_1^2 \left(t^2 - 7t + 10\right) dt - \int_2^5 \left(t^2 - 7t + 10\right) dt + \int_5^7 \left(t^2 - 7t + 10\right) dt$$

If you are allowed to use your calculator, you use FnInt(abs(X^2-7X+10),X,1,7)

Example 4) Given an object moving in a straight line with $a(t)=\sqrt{t}\ \frac{\text{ft}}{\sec^2}$, $v(0)=-18$, $t=0$ to $t=16$, find $v(t)$ and the displacement and distance of the object.

$v(t) = \int t^{1/2}\,dt$

$v(t) = \frac{2}{3}t^{3/2} + C$

$v(0) = C = -18$

So, $v(t) = \frac{2}{3}t^{3/2} - 18$

disp $= \int_0^{16} \left(\frac{2}{3}t^{3/2} - 18\right) dt$

dist $= \int_0^{16} \left|\frac{2}{3}t^{3/2} - 18\right| dt$

$= -\int_0^{9} \left(\frac{2}{3}t^{3/2} - 18\right) dt$

$+ \int_9^{16} \left(\frac{2}{3}t^{3/2} - 18\right) dt$

Example 5) A subway train accelerates as it leaves one station, then decelerates as it comes into the next station. following chart measures the velocity v given in miles per hour .

time t	0	5	10	15	20	25	30	35	40	45	50	55	60
velocity	0	15	33	42	44	44	44	44	43	38	24	8	0
Distance													

a) find the distance the train travels every 5 second interval.
b) find the total distance between subway stops.

Given the velocity of an object in ft/sec, find the displacement and distance traveled in the given time interval.

1. $v(t) = 12 - 3t \quad [0, 5]$
2. $v(t) = t^2 - 10t + 16 \quad [0, 6]$

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Given the velocity of an object in ft/sec, find the displacement and distance traveled in the given time interval.

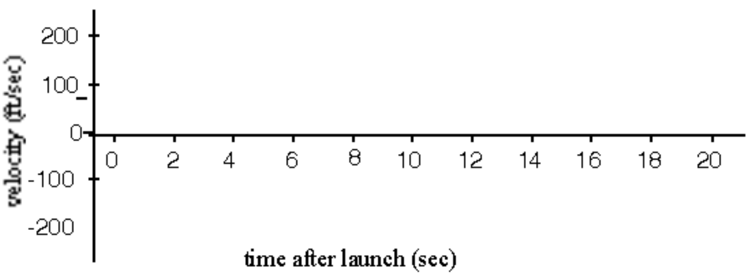
1. $v(t) = 12 - 3t$ [0, 5]
2. $v(t) = t^2 - 10t + 16$ [0, 6]

Given the acceleration of an object in ft/sec^2 and its initial velocity, find the displacement and distance traveled in the given time interval.

3. $a(t) = 4t, v(0) = -8$ [0, 3]

4. $a(t) = 6\sin t, v(0) = -9$ [0, π]

5) When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout the rocket coasts upward for awhile and then begins to fall. A small explosive charge pops out a parachute while the rocket is on its way down. The parachute slows the rocket to keep it from braking when it lands (after 20 seconds). The velocity function is given below. Using your calculator, sketch it. Questions a through f should be answered by looking at your graph. Question g should be calculated.



$$f(t)=\begin{cases}50t^2, t \le 2\\ 260-30t, 2 < t \le 15\\ -7.6(t-20)^2, t > 15\end{cases}$$

a. For how many seconds did the engine burn? _______
b. How fast was the rocket going when the engine stopped? _______
c. When did the rocket reach its highest point? _______
d. When did the parachute pop out? ________
e. How long did the rocket fall before the parachute opened? ______
f. How fast was the rocket falling when the parachute opened? _______
g. When was the rocket's acceleration the greatest? ________

h. What was the total distance the rocket traveled? __________

Straight Line Motion - Revisited - Homework

Given the velocity of an object in ft/sec, find the displacement and distance traveled in the given time interval

1. $v(t) = 12 - 3t \quad [0, 5]$

$$\text{Disp} = \int_0^5 (12 - 3t)\, dt = 22.5 \text{ ft}$$

$$\text{Dist} = \int_0^5 |12 - 3t|\, dt$$

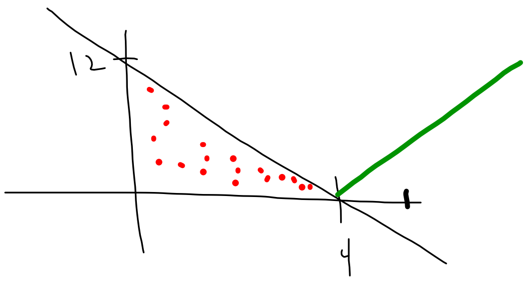
$$\text{Dist} = \int_0^4 (12 - 3t)\, dt - \int_4^5 (12 - 3t)\, dt = 25.5 \text{ ft}$$

2. $v(t) = t^2 - 10t + 16 \quad [0, 6]$

$$\text{Disp} = \int_0^6 (t^2 - 10t + 16)\, dt = -12 \text{ ft}$$

$$\text{Dist} = \int_0^6 |t^2 - 10t + 16|\, dt$$

$$\text{Dist} = \int_0^2 (t^2 - 10t + 16)\, dt - \int_2^6 (t^2 - 10t + 16)\, dt = 41.333 \text{ ft}$$



Given the acceleration of an object in ft/sec^2 and its initial velocity, find the displacement and distance in the given time interval

3. $a(t) = 4t, v(0) = -8 \quad [0,3]$

$v(t) = 2t^2 - 8$

$\text{Disp} = \int_0^3 (2t^2 - 8)\, dt = -6 \text{ ft} = \left(\frac{2t^3}{3} - 8t\right)\Big|_0^3 = (18 - 24) - 0$

$= -6$

$\text{Dist} = \int_0^3 |2t^2 - 8|\, dt$

$\text{Dist} = -\int_0^2 (2t^2 - 8)\, dt + \int_2^3 (2t^2 - 8)\, dt = 15.333 \text{ ft}$

4. $a(t) = 6\sin t, v(0) = -9 \quad [0,\pi]$

$v(t) = -6\cos t + C, C = -3, v(t) = -6\cos t - 3$

$\text{Disp} = \int_0^\pi (-6\cos t - 3)\, dt = -9.425 \text{ ft}$

$\text{Dist} = \int_0^\pi |-6\cos t - 3|\, dt = 13.534 \text{ ft}$