Do Now: Review (from September)

11. If $s(t) = t^3 + t - 1$ is a measure of feet traveled ~~per second~~, find

a) the average velocity between $t = 2$ and $t = 7$

b) the instantaneous velocity at $t = 2$ seconds.

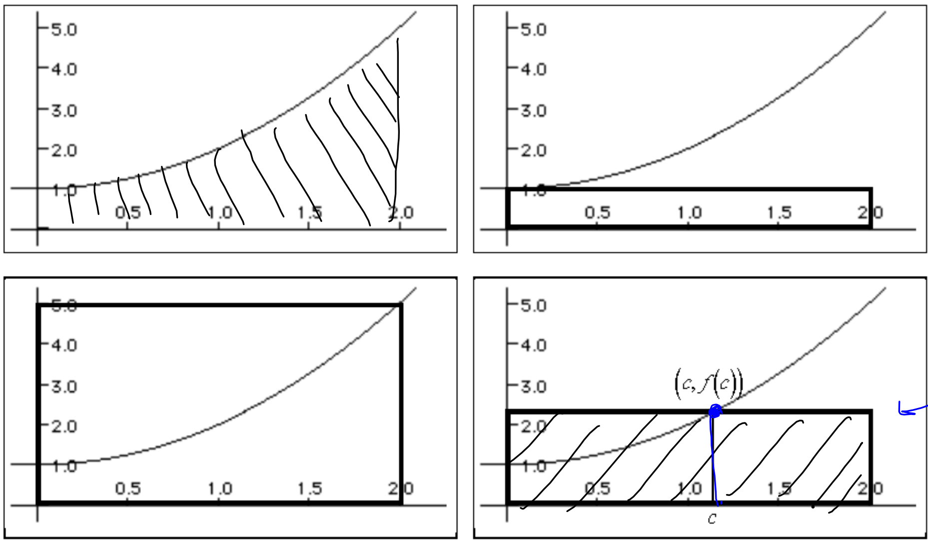
$$\frac{s(7) - s(2)}{7-2} = \text{ave vel}$$

$$s'(2) = \_\_\_\_\_\_\_\_$$

p163

Average Value of a Function & 2nd Fund. Thm. - Classwork

On the graph below left, $f(x)$, we know that the area under the curve between 0 and 2 is $\int_0^2 f(x)\,dx$. The question is, is there a rectangle whose base is 2 which is exactly equal to this integral? Clearly, the area under the rectangle below right is smaller than the area under $f(x)$.



Clearly, the area under the rectangle is greater than the area under the curve $f(x)$.

The area under the curve in this picture is now fairly close to the area under the rectangle.

height of this rect. is the average value of f(x) on [0,2]

This can be summarized by the **Mean Value Theorem for Integrals** (not to be confused with the MVT for derivatives).

If f is continuous on the closed interval $[a,b]$, then there exists a number c on $[a,b]$ such that $\boxed{\int_a^b f(x)\,dx = f(c)(b-a)}$. What this says is that the area under the curve between a and b can be expressed as the base of the rectangle $(b-a)$ times the height of the rectangle at some point $(c, f(c))$.

Since $\int_a^b f(x)\,dx = f(c)(b-a)$, we can write $\frac{\int_a^b f(x)\,dx}{b-a} = \boxed{f(c)}$. This $\frac{\int_a^b f(x)\,dx}{b-a}$ is called the average value of a function. It represents the height of the rectangle guaranteed by the MVT for integrals.

a b

Example 1) Given $f(x) = x^2 + 1$ on the interval [0, 2] (the problem on the previous page), find

a) the average value of the function

b) the value of c guaranteed by the MVT for integrals.

$$f(c) = \frac{1}{b-a}\int_0^2 (x^2+1)\,dx$$

$$= \frac{1}{2}\left(\frac{x^3}{3} + x\right)\Big|_0^2$$

$$= \frac{1}{2}\left(\frac{8}{3} + 2\right)$$

$$= 7/3$$

ave value or height of rect.

$$f(x) = x^2 + 1 = 7/3$$

$$x^2 = 4/3$$

$$c = x = 2/\sqrt{3}$$

Example 2) Given $f(x) = \sin x$ on the interval $[0, \pi]$, find

Example 2) Given $f(x) = \sin x$ on the interval [0, π], find

a) the average value of the function

$$f(c) = \frac{1}{\pi}\int_0^{\pi} \sin x \, dx$$

$$= -\frac{1}{\pi}(\cos x)\Big|_0^{\pi}$$

$$= -\frac{1}{\pi}(-1 - 1)$$

$$= 2/\pi$$

b) the value of c guaranteed by the MVT for integrals.

$$\sin x = \frac{2}{\pi}$$

$$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{2}{\pi}\right)$$

$$x = .690$$

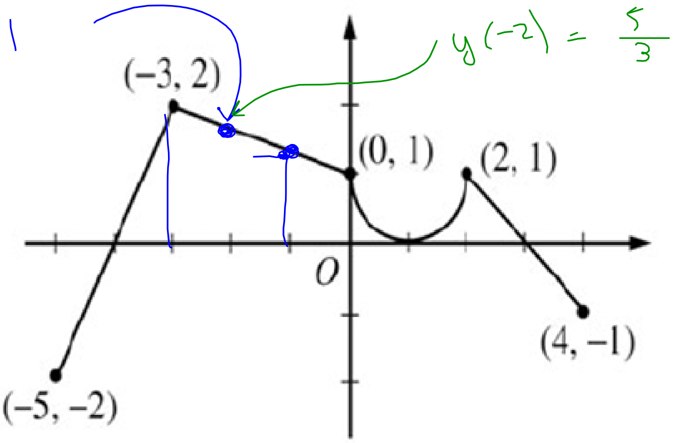
Suppose you were asked to find $\int_1^x (t^2 - 4t + 1)\,dt$. Use the Fundamental Theorem of Calculus to do so.

Now, take the derivative of your answer. ________________ . What is the basic result? ___________

$y = -\frac{1}{3}x + 1$

$y(-1) = \frac{4}{3}$

$y(-2) = \frac{5}{3}$



Graph of f

$$\int_{-3}^{-1} f(t)\,dt = \frac{1}{2}\left(2 + \frac{4}{3}\right)2 = 10/3$$

$$\int_{-2}^{0} f(t)\,dt = \frac{1}{2}\left(\frac{5}{3} + 1\right)2 = 8/3$$

Norristown Area High School (393020) - Statistics, All Students

Section Summary								Subject Summary	
Score	5	4	3	2	1	Total Students	Average Score	Total Students	Average Score
Total Students	2	4	7	1	6	**20**	**2.750**	**20**	**2.750**

Student Name	Student Identifier	AP Number	Score	
	260448	42099520	1	
	250382	42076988	3	F Bell
	270224	42099571	5	V Cook
	310419	42099732	4	J Culp
	230318	42099546	3	N Danko
	230464	42077011	1	
	220110	42077364	1	
	220142	42075485	1	
	241354	42075698	1	
	200353	42075647	3	C Kinney
	220072	42077101	5	B Nguyen
	310310	42099431	3	H Olafsen
	19401	42099660	3	E Palesano
	220045	42099635	4	K Patel
	220125	42099341	3	J Pigeon
	270245	42099651	4	M Purcell
	280412	42077127	3	W Romano
	310306	34692963	4	J Schinel
	220033	34692955	2	
	211285	34693013	1	

p164

Suppose you were asked to find $\int_{1\,0}^{x}\left(t^2-4t+1\right)dt$. Use the Fundamental Theorem of Calculus to do so.

Now, take the derivative of your answer. x^2-4x+1. What is the basic result? int ; diff undo each other.

$$= \left(\frac{t^3}{3}-2t^2+t\right)\Big|_{1\,0}^{x}$$

$$= \frac{x^3}{3}-2x^2+x + \left(\cancel{+}\ \boxed{-\tfrac{2}{3}}\ \right)$$

The Second Fundamental Theorem of Calculus.

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

What this says that the constant on the bottom doesn't matter when we take the derivative of an integral. The derivative of the integral is the original integrand (but with the variable changed).

baby $= x^2$

Example 3) Find the following.

a. $\frac{d}{dx}\left[\int_1^x \sqrt{t^2-1}\,dt\right]$

$= \sqrt{x^2-1}$

b. $\frac{d}{dx}\left[\int_x^3 t\sin t\,dt\right]$

$= \frac{d}{dx}\left[-\int_3^x t\sin t\,dt\right]$

$= -x\sin x$

c. $\frac{d}{dx}\left[\int_{\pi/2}^{x^2} \cos t\,dt\right]$

mom $= \frac{d}{dx}\int_{\pi/2}^{x} \cos t\,dt$

$= \cos(x^2)\cdot 2x$

$= 2x\cos(x^2)$

$$\frac{d}{dx}\int_{-1}^{\sqrt{x}} \frac{e^t}{2}\,dt = \frac{e^{\sqrt{x}}}{2}\cdot\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{e^{\sqrt{x}}}{2}\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{4\sqrt{x}}$$

$$\frac{d}{dx}\int_{1/x}^{-2} \csc^2(t^2-1)\,dt =$$

$$= \frac{d}{dx}\left[-\int_{-2}^{1/x} \csc^2(t^2-1)\,dt\right]$$

$B = 1/x$

$$M = \frac{d}{dx}\left[-\int_{-2}^{x} \csc^2(t^2-1)\,dt\right] = -\csc^2\left(\left(\frac{1}{x}\right)^2-1\right)\cdot\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= \frac{\csc^2\left(\frac{1}{x^2}-1\right)}{x^2}$$

Average Value of a Function & 2nd Fund. Thm. - Homework

1. Find the average value of the function f on the given interval. Do not use calculators.

a) $f(x) = x^2 - 2x$ [0, 3]

b) $f(x) = \cos x$ $\left[0, \frac{\pi}{2}\right]$