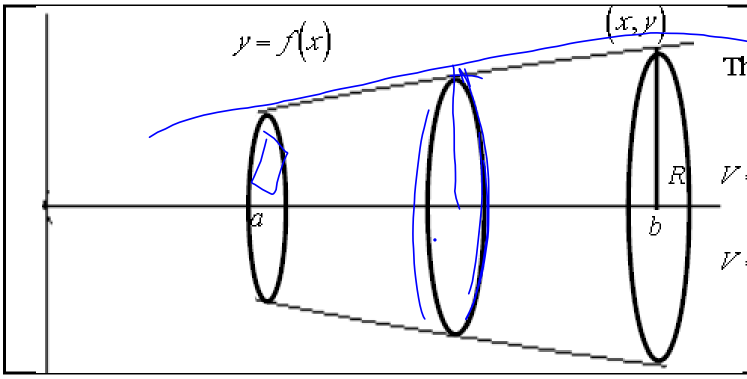
Volume by Disk/Washers - Classwork

1st problem: You are given the function $y = f(x)$ and you are looking at between $x = a$ and $x = b$. You are then going to rotate the function about the x-axis. The 3-D object you get appears like the one below. We wish to find the volume of the object. If you are to look at a cross section by slicing it perpendicular to the x-axis, we get a circle. The formal name for this circle is **a disk**. The area of this disk is πR^2 where R is the radius of the disk. In reality though, this disk is a 3-D shape with a width of Δx. So the volume of a representative disk is $\pi R^2 \cdot \Delta x$. If we take thinner and thinner disks, Δx gets close to zero. So the volume of the solid is $\lim_{n \to \infty} \sum_{i=1}^{n} \pi R_i^2 \cdot \Delta x$ which we know to be a Reimann sum. Our job is to find R. The way we will do this is to draw the Radius and label the endpoint as (x, y) So we end up with the disk formula: $V = \pi \int_a^b R^2 dx$

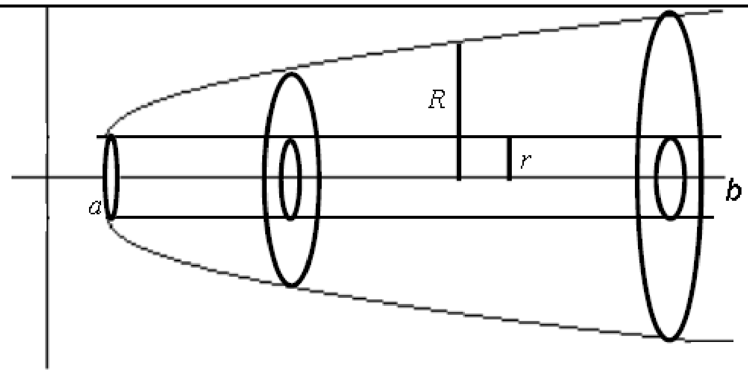


The Disk Formula

$V = \pi \int_a^b R^2 dx$ if width of disk is along x - axis

$V = \pi \int_c^d R^2 dy$ if width of disk is along y - axis

2nd problem: The problem remains the same except the region between two curves are rotated about the x-axis. Instead of creating a disk, we create a washer (2 concentric circles with "air" in between) Using the same argument, the area of the washer is $R^2 - r^2$ where R is the outside radius and r is the inside radius. So, we end up with $V = \pi \int_a^b \left(R^2 - r^2\right) dx$



The Washer Formula

$V = \pi \int_a^b \left(R^2 - r^2\right) dx$ if width of washer is along x - axis

$V = \pi \int_c^d \left(R^2 - r^2\right) dy$ if width of washer is along y - axis

Generalizations:

1. Draw the function (s). 2. Determine the region to rotate. **3a**. If it is rotated about the x-axis or any horizontal line, it is a dx problem and everything must be in terms of x. **3b** If it is rotated about the y-axis or any vertical line, it is a dy problem and everything must be in terms of y. **4a** If it is a disk problem, you need to find R. **4b** . If it is washer problem, you need to find R and r. **5**. When in doubt, label the point on the disk/washer as (x,y). y is a vertical distance and x is a horizontal distance.

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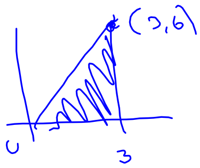
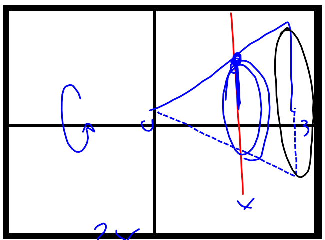


Example 1) Find the volume if the region enclosing $y = 2x, y = 0, x = 3$ is rotated about the

a) x-axis



R = $2x$ r = ______

V = ______

$V = \pi \int R^2$

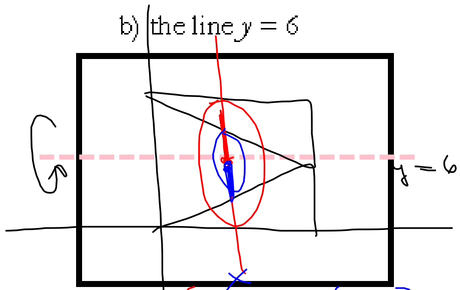
$V = \pi \int_0^3 (2x)^2 \, dx$

$V = \pi \cdot \frac{4x^3}{3} \Big|_0^3$

$= \pi (36 - 0)$

$= 36\pi$

b) the line $y = 6$



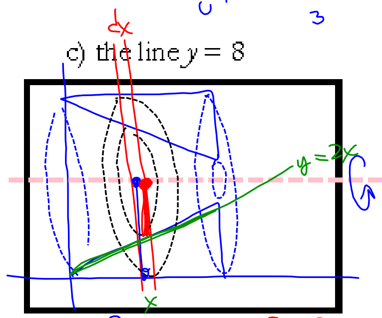
R = 6 r = $6 - 2x$

V = ______

$\pi \int (R^2 - r^2) \, dx$

$\pi \int_0^3 \left(6^2 - (6 - 2x)^2\right) dx$

c) the line $y = 8$



R = 8 r = $8 - 2x$

V = ______

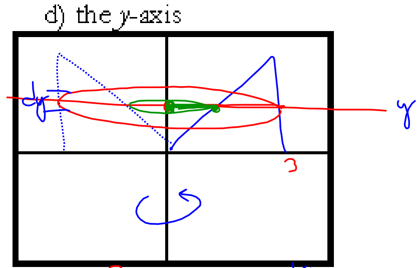
$\pi R^2 - \pi r^2$

$\pi \int_0^3 (R^2 - r^2) \, dx$

$x = \frac{1}{2}y$

Example 1) Find the volume if the region enclosing $y = 2x$, $y = 0$, $x = 3$ is rotated about the

d) the y-axis

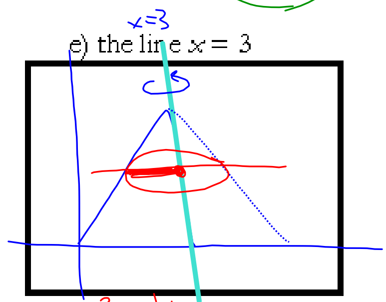


R = 3 r = $\frac{1}{2}y$

V = ____________

$$\pi \int_{y=0}^{6} \left(3^2 - \left(\frac{y}{2}\right)^2 \right) dy$$

e) the line $x = 3$

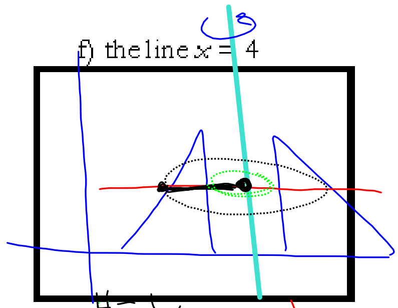


R = $3 - \frac{1}{2}y$ r = ____________

V = ____________

$$\pi \int_{y=0}^{6} \left(3 - \frac{1}{2}y\right)^2 dy$$

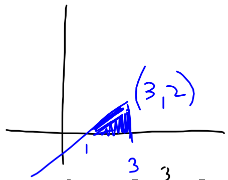
f) the line $x = 4$



R = $4 - \frac{1}{2}y$ r = 1

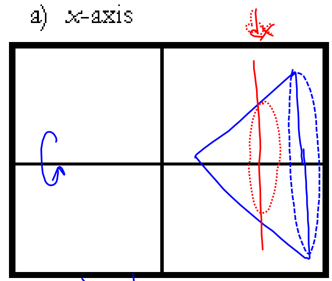
V = ____________

$$\pi \int_{y=0}^{6} \left(\left(4 - \frac{y}{2}\right)^2 - 1 \right) dy$$



Example 2) Find the volume if the region enclosing $y = x - 1, y = 0, x = 3$ is rotated about the

a) x-axis

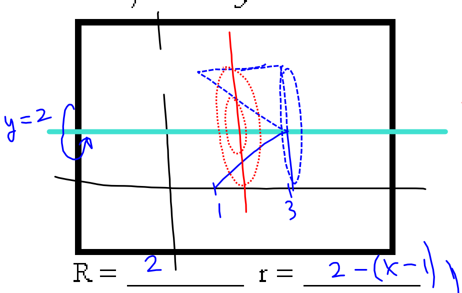


R = $x-1$ r = ______

V = ______

$$\pi \int_1^3 (x-1)^2 \, dx$$

b) the line $y = 2$



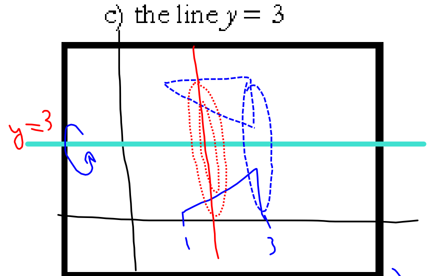
R = 2 r = $2-(x-1)$

V = ______

$r = 3-x$

$$\pi \int_1^3 \left(2^2 - (3-x)^2\right) dx$$

c) the line $y = 3$



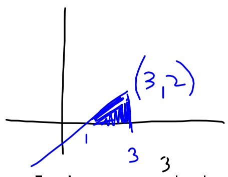
R = 3 r = $3-(x-1)$

V = ______

$r = 4-x$

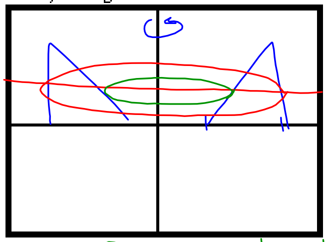
$$\pi \int_1^3 \left(3^2 - (4-x)^2\right) dx$$

$x = y+1$



Example 2) Find the volume if the region enclosing $y = x - 1, y = 0, x = 3$ is rotated about the

d) the y-axis

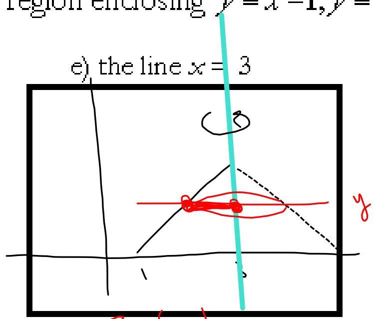


R = 3 r = $y+1$

V = ____

$$\pi\int_{y=0}^{2}\left(3^2-(y+1)^2\right)dy$$

e) the line $x = 3$

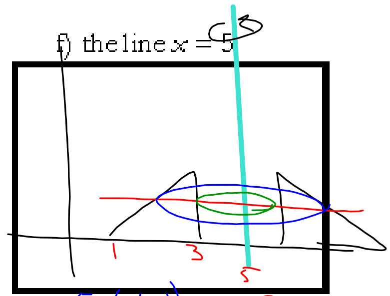


R = $3-(y+1)$ r = ____

V = ____

$$\pi\int_{y=0}^{2}(2-y)^2\,dy$$

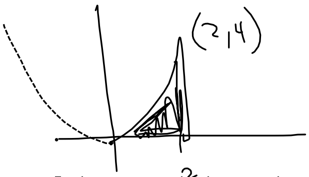
f) the line $x = 5$



R = $5-(y+1)$ r = 2

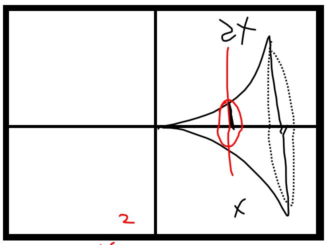
V = ____

$$\pi\int_{y=0}^{2}\left((4-y)^2-2^2\right)dy$$



Example 3) Find the volume if the region enclosing $y = x^2, y = 0, x = 2$ is rotated about the

a) x-axis



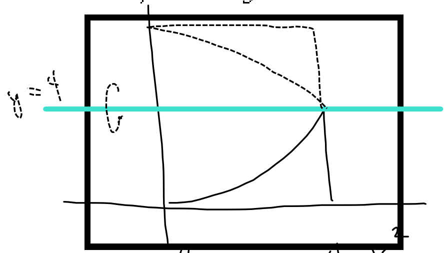
R = x^2 r = ______

V = ______

$$\pi \int_0^2 (x^2)^2 \, dx$$

$$= \pi \int_0^2 x^4 \, dx$$

b) the line $y = 4$



R = 4 r = $4 - x^2$

V = ______

$$\pi \int_0^2 \left(4^2 - (4 - x^2)^2\right) dx$$

c) the line $y = 5$

R = 5 r = $5 - x^2$

V = ______

$$\pi \int_0^2 \left(5^2 - (5 - x^2)^2\right) dx$$

Example 3) Find the volume if the region enclosing $y = x^2, y = 0, x = 2$ is rotated about the

d) the y-axis

R = ________ r = ________

V = ______________________

e) the line $x = 2$

R = ________ r = ________

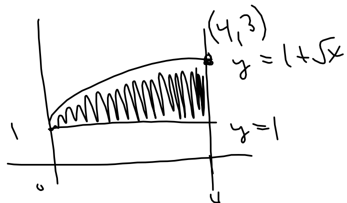
V = ______________________

f) the line $x = 4$

R = ________ r = ________

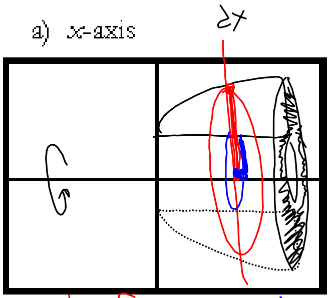
V = ______________________

$\pi R^2 - \pi r^2$



Example 4) Find the volume if the region enclosing $y = 1+\sqrt{x}, y = 1, x = 4$ is rotated about the

a) x-axis

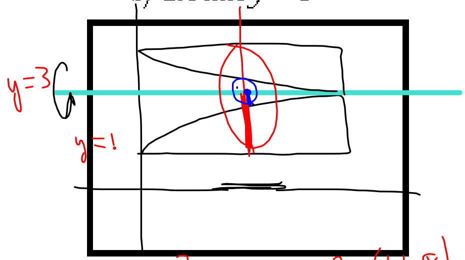


R = $1+\sqrt{x}$ r = 1

V =

$$\pi \int_0^4 \left((1+\sqrt{x})^2 - 1^2 \right) dx$$

b) the line $y = 3$



R = 2 r = $3-(1+\sqrt{x})$

V =

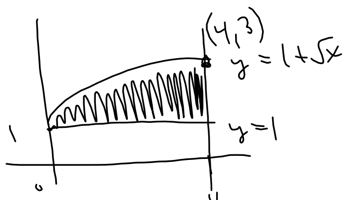
$$\pi \int_0^4 \left(2^2 - (2-\sqrt{x})^2 \right) dx$$

c) the line $y = 5$

R = $5-1=4$ r = $5-(1+\sqrt{x})$

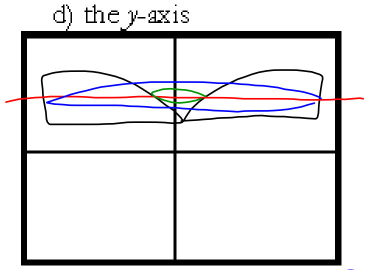
V =

$x = (y-1)^2$



Example 4) Find the volume if the region enclosing $y = 1+\sqrt{x}, y = 1, x = 4$ is rotated about the

d) the y-axis

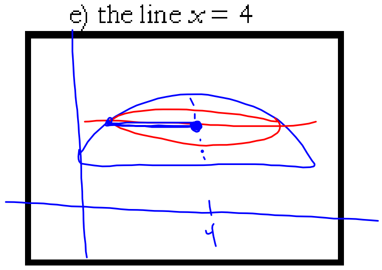


R = 4 r = $(y-1)^2$

V =

$$\pi \int_{y=1}^{3} \left(4^2 - \left((y-1)^2\right)^2\right) dy$$

e) the line $x = 4$

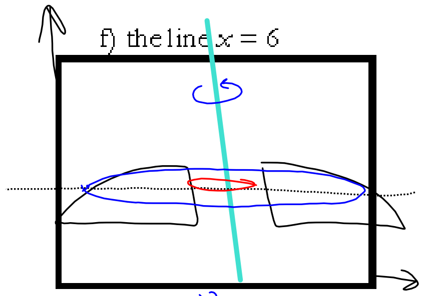


R = ______ r = ______

V =

$$\pi \int_{y=1}^{3} \left(4 - (y-1)^2\right)^2 dy$$

f) the line $x = 6$



R = $6-(y-1)^2$ r = 2

V =

$$\pi \int_{y=1}^{3} \left[\left(6-(y-1)^2\right)^2 - 2^2\right] dy$$