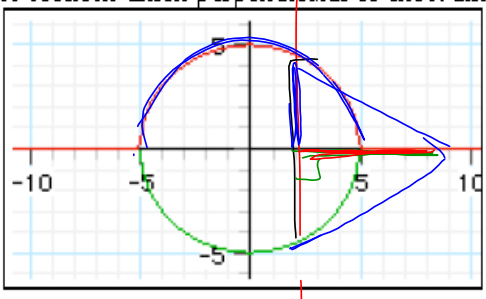
Example 8) Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 25$ with the indicated cross sections taken perpendicular to the x-axis



$y = \sqrt{25 - x^2}$

a) squares

b) equilateral triangles

$A_\square = \left[2\sqrt{25-x^2}\right]^2$

$A_\square = 4(25 - x^2)$

$V = \int_{x=-5}^{5} A_\square \, dx = 4\int_{-5}^{5} (25 - x^2)\, dx$

$= \frac{2000}{3}$

$A_\triangle = \frac{1}{2}bh$

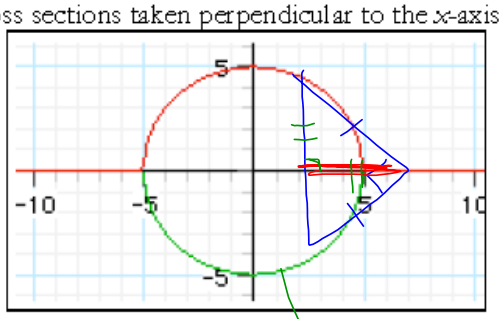
$= \frac{1}{2}\left(2\sqrt{25-x^2}\right)\cdot\sqrt{3}\cdot\sqrt{25-x^2}$

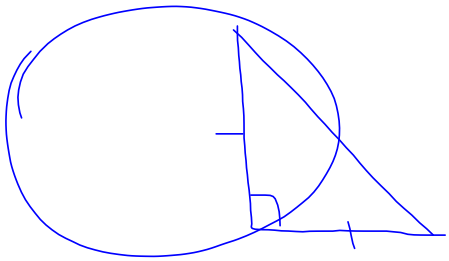
$= \sqrt{3}(25 - x^2)$

$V = \sqrt{3}\int_{-5}^{5} (25 - x^2)\, dx$

$= \frac{500\sqrt{3}}{3}$

Example 8) Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 25$ with the indicated cross sections taken perpendicular to the x-axis





a) squares

b) equilateral triangles

a) semicircles

b) isosceles right

$$A_D = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\pi\left(\sqrt{25-x^2}\right)^2$$

$$= \frac{\pi(25-x^2)}{2}$$

$$V = \frac{\pi}{2}\int_{-5}^{5}(25-x^2)\,dx$$

$$= \frac{\pi}{2}\cdot\frac{500}{3}$$

$$= \frac{250\pi}{3}$$

$$A_D = \frac{1}{2}bh$$

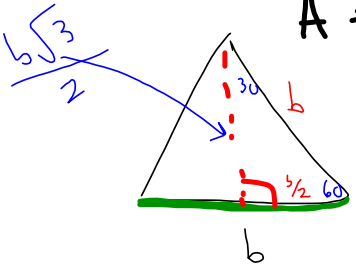
$$= \frac{1}{2}\cdot 2\sqrt{25-x^2}\cdot\sqrt{25-x^2}$$

$$= (25-x^2)$$

$$V = \int_{-5}^{5}(25-x^2)\,dx = \frac{500}{3}$$

do now Find the area of each:

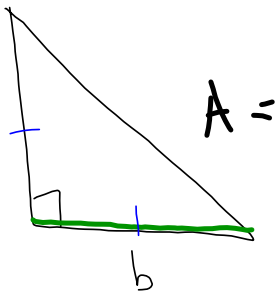
① Equilateral triangle



$A = \frac{b^2\sqrt{3}}{4}$

a)

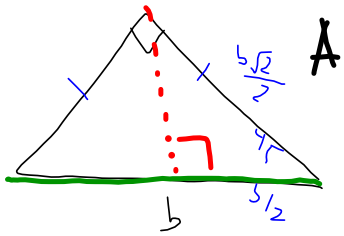
② Isoceles Right



$A = \frac{1}{2}b^2$

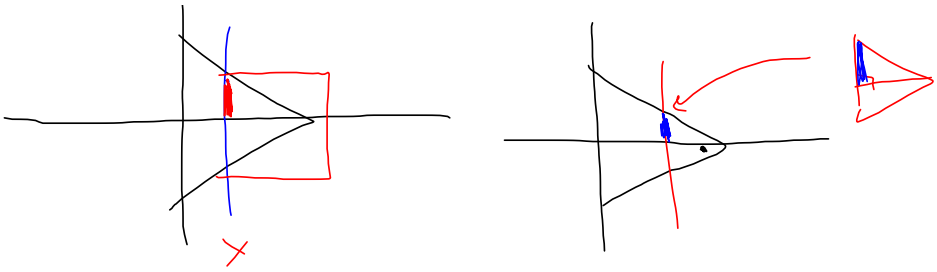
b)

Isoceles Right



$A = \frac{b^2}{4}$

$\frac{1}{2} \cdot b \cdot \frac{b}{2}$



Example 9) Find the volume of the solid whose base is bounded by the lines $y = x - 4, y = 4 - x$ and $x = 0$ with the indicated cross sections taken perpendicular to the x-axis

a) squares

$A_{\square} = s^2 = [2(4-x)]^2$

$= 4(4-x)^2$

$V = 4\int_0^4 (4-x)^2\,dx$

b) equilateral triangles

$A = \frac{1}{2}bh$

$= \frac{\sqrt{3}}{2}\cdot(4-x)\cdot\sqrt{3}\cdot(4-x)$

$V = \sqrt{3}\int_0^4 (4-x)^2\,dx$

c) semi circles

d) isosceles right triangles