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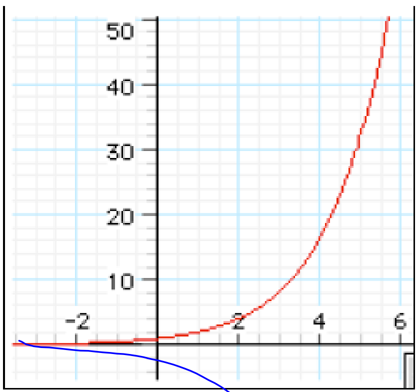
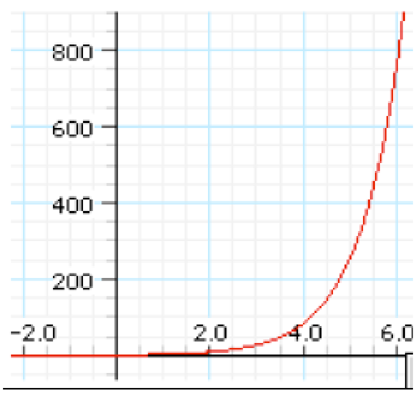
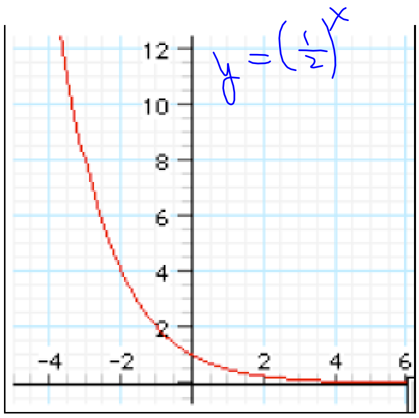
Review of Exponentials and Logarithms - Classwork

In our study of calculus, we have examined derivatives and integrals of polynomial expressions, rational expressions, and trignometric expressions. What we have not examined are exponential expressions, expressions of the form $y = a^x$. While these are covered extensively in precalculus, a little review is in order and these types of expressions are very prevalent in the calculus theatre.

An expressions in the form of $y = a^x$ will graph an exponential. An exponential graph tends to "explode" based on the value of x, since the x is in the exponent.

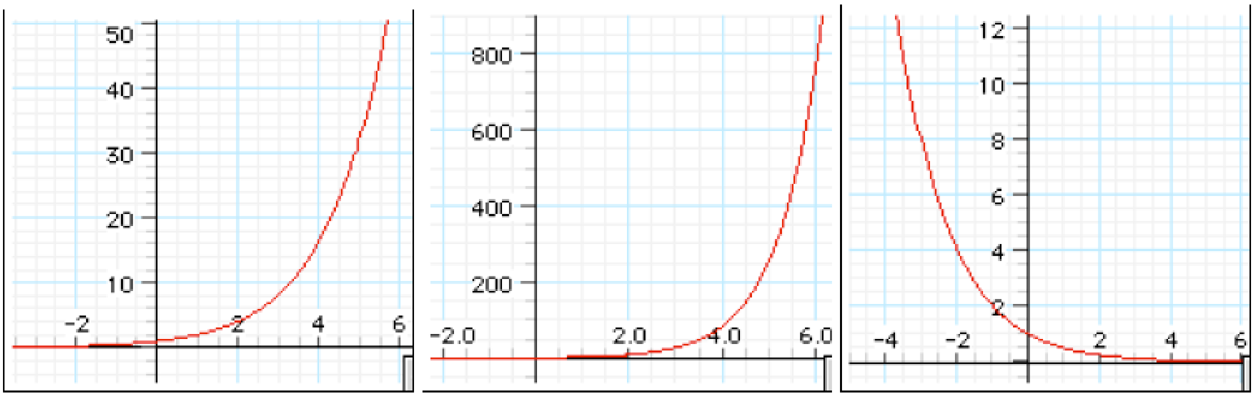






$y = 2^x$ $y = 3^x$ $y = .5^x$

$y = -2^x$



$y = 2^x$ $y = 3^x$ $y = .5^x$

The graph of various exponential curves are shown above. We know that the graph of $y = 1^x$ graphs y = 1.

If $a < 0$, the graph of $y = a^x$ will not exist at certain points for instance, what is $(-2)^{1/2}$? DNE

So it only makes sense to examine functions in form of $y = a^x$, if $a > 0$, $a \neq 1$. When $a > 1$, we get what is called a growth curve and the larger a is, the steeper the growth curve is. If $0 < a < 1$, the we get a decay cuvre as shown in the 3rd graph above. No matter what, exponential curves in the form of $y = a^x$ have certain features.

What point do they have in common? (0, 1) What is the domain? $\mathbb{R}$ What is the range? $y > 0$

Solving basic exponential equations can be accomplished by using the fact that if $a^x = a^y$, then $x = y$

Examples) Solve for x:

1) $2^{x+1} = 8$

$2^{x+1} = 2^3$

$x+1 = 3$

$x = 2$

2) $3^{2x-3} = \frac{1}{3}$

$3^{2x-3} = 3^{-1}$

$2x - 3 = -1$

$2x = 2$

$x = 1$

3) $4^{5x-1} = \sqrt[3]{32}$

$(2^2)^{5x-1} = (2^5)^{1/3}$

$2(5x-1) = 5/3$

$10x - 2 = 5/3$

$10x = 11/3$

$x = 11/30$

4) $7^{2x+3} = \left(\frac{1}{49}\right)^{3-x}$

$7^{2x+3} = (7^{-2})^{3-x}$

$\cancel{2x}+3 = -6 + \cancel{2x}$

no solution

5) $8^{5-2x} = 1$

$8^{5-2x} = 8^0$

$x = 2.5$

Solving basic exponential equations can be accomplished by using the fact that if $a^x = a^y$, then $x = y$

Examples) Solve for x:

1) $2^{x+1} = 8$ 2) $3^{2x-3} = \frac{1}{3}$ 3) $4^{5x-1} = \sqrt[3]{32}$ 4) $7^{2x+3} = \left(\frac{1}{49}\right)^{3-x}$ 5) $8^{5-2x} = 1$

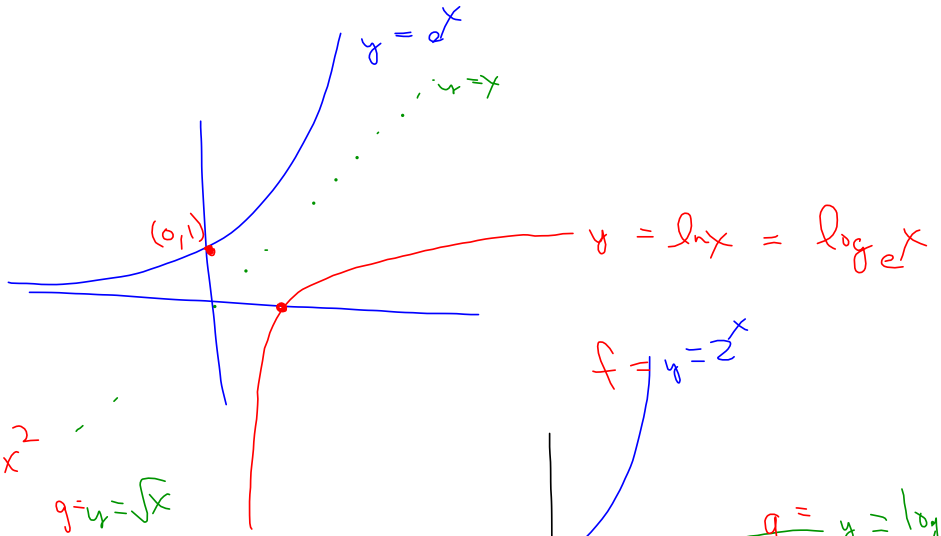
Solving exponential equations like the ones above are easy when each side of the equation have common bases. But problems like $3^{x-1} = 4$ cause problems. With that problem created, we introduced the concept of logarithms. A logarithm is simply an inverse of an exponential. Students typically hear the word logarithm and go into a cold sweat because they do not understand them. So lets get it straight once and for all.

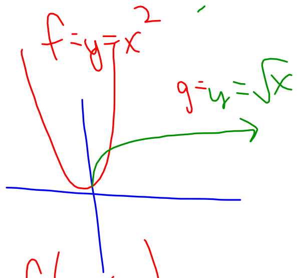
The statement $y = b^x$ can be written in an alternate way: $x = \log_b y$. They mean the same thing.

Whenever you are given a logarithmic statement, write it exponentially. You will know the answer!

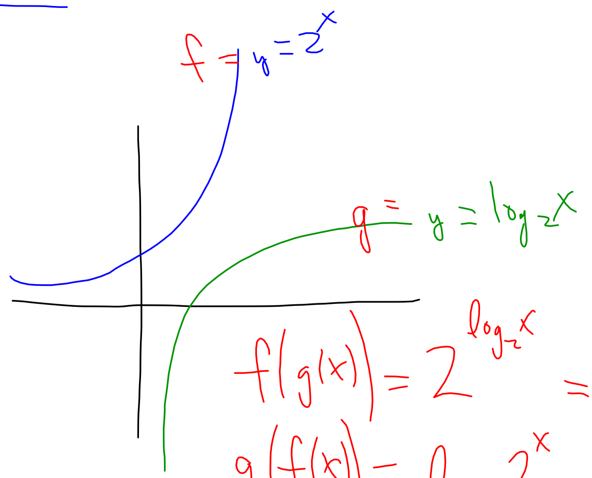
Examples: find the value of the following:

1) $\log_2 8$ 2) $\log_5 \frac{1}{25}$ 3) $\log_8 \sqrt{2}$ 4) $5\log_6 1$ 5) $\log_7 0$





$$f(g(x)) = x$$



$$f(g(x)) = 2^{\log_2 x} = x$$

$$g(f(x)) = \log_2 2^x = x$$

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Examples: find the value of the following:

1) $\log_2 8 = 3$

$8 = 2^x$

$x = 3$

2) $\log_5 \frac{1}{25} = -2$

$5^x = \frac{1}{25}$

$x = -2$

3) $\log_8 \sqrt{2} = 1/6$

$8^x = 2^{1/2}$

$(2^3)^x = 2^{1/2}$

$3x = 1/2$

$x = 1/6$

4) $5\log_6 1 =$

$= 5 \cdot \underbrace{\log_6 1}_{0}$

$= 5 \cdot 0$

$= 0$

5) $\log_7 0$ DNE

$7^x = 0$

$x = ?$

If the base is not specified, it is assumed to be 10. $\log_{10} x$ and $\log x$ are the same things.

Examples) Find the value of the following

6) $\log 100$ 7) $\log 1$ $= 0$ 8) $5\log\sqrt{10}$ 9) $\log\frac{1}{1000}$ $= -3$ 10) $\log 10^{\sqrt{5}}$ $= \sqrt{5}$

$= \log_{10} 100$

$= 2$

$= 5 \cdot \log_{10} 10^{1/2}$

$= 5 \cdot 1/2$

$= 5/2$

$10^{x} = 10^{\sqrt{5}}$

$\log_{10}\left(10^{\sqrt{5}}\right) = \sqrt{5}$

On what seems to be a side note, let's examine the expression $y = \left(1+\frac{1}{x}\right)^{x}$ for various values of x. What we are doing is looking at $\left(1+\frac{1}{2}\right)^{2}, \left(1+\frac{1}{3}\right)^{3}, \left(1+\frac{1}{4}\right)^{4}, \ldots \left(1+\frac{1}{50}\right)^{50} \ldots$ Logic would tell us that as x gets larger, $1+\frac{1}{x}$ gets closer to 1 and thus the limit as x approaches infinity of $1+\frac{1}{x}$ is 1. Thus, logic also dictates that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} =$ 1. But when you play with infinity, logic doesn't always work. You can see that if you set up your calculator with the expression Y1 = $\left(1+\frac{1}{x}\right)^{x}$ and look at a table of values.

x	$\left(1+\frac{1}{x}\right)^{x}$	x	$\left(1+\frac{1}{x}\right)^{x}$	x	$\left(1+\frac{1}{x}\right)^{x}$	x	$\left(1+\frac{1}{x}\right)^{x}$
1	2	100	2.70481383	1000	2.71692393	100000	2.71826824
2	2.25	110	2.70602808	2000	2.71760257	200000	2.71827503
3	2.37037037	120	2.70704149	3000	2.71782892	300000	2.7182773
4	2.44140625	130	2.70790008	4000	2.71794212	400000	2.71827843
5	2.48832	140	2.70863681	5000	2.71801005	500000	2.71827911
6	2.52162637	150	2.70927591	6000	2.71805534	600000	2.71827956
7	2.5464997	160	2.70983558	7000	2.71808769	700000	2.71827989
8	2.56578451	170	2.71032975	8000	2.71811196	800000	2.71828013
9	2.58117479	180	2.7107693	9000	2.71813083	900000	2.71828032
10	2.59374246	190	2.71116279	10000	2.71814593	1000000	2.71828047

$e \approx$

pronounced "oiler's"

If should be obvious that as x gets larger and larger, the expression $1+\frac{1}{x}$ is not approaching one but the number ______ e ______. This number is a very special number in mathematics and is called Euler's number. Leonhard Euler (1707 - 1783) discovered this number and it is known as e. The value of e is 2.718281828.... e is a transcendental number which, like π and $\sqrt{2}$, continues on forever without any pattern. (Note: the 1828 in e, although appearing twice consecutively near the start does not appear again for a very long while. It is completely coincidental that it appears twice)

~~π~~ ~~4~~ ~~8~~ ~~2~~ 10

The number e is such an important number (if you would have to decide what the 5 most important numbers are, what would they be? 10, π, e, i, 0), that it forms the basic of what are called natural logarithms of Napierian logs (after John Napier, 1550-1617, who first used them). Just as logarithms (log) use base 10, natural logs (ln) use base e. When you wish to find the value of a log, you write the expression exponentially. You do the same thing with a natural log except that your base is now e.

$\log_e 10$

For instance, to find ln 10, you call it x, and are now solving the equation $e^x = 10$. Since e is slightly below 3, we expect ln 10 to be between the values of 2 and 3.

So, given the function $y = e^x$, the domain is $\mathbb{R}$ and the range is $y > 0$.

Examples) Find the value of the following

11) $\ln e^4 = 4$

12) $9 \ln 1$
$= 9 \cdot 0$
$= 0$

13) $8 \ln \sqrt{e}$
$= 8 \cdot 1/2$
$= 4$

14) $\ln \frac{1}{e^3} = -3$
$e^x = e^{-3}$

15) $e^{\ln 5} = 5$
$e^x \leftrightarrow \ln x$
$e^{\ln x} = x$

There are three basic rules for operation with logarithms that you must know. They are as follows:

1. $\log(a \cdot b) = \log a + \log b$	2. $\log\left(\dfrac{a}{b}\right) = \log a - \log b$	3. $\log a^b = b \log a$

These rules work with logs to any base of the ln function.

Examples) Find the value of the following expressions:

12) $\log 25 + \log 4$ 12) $\log_2 40 - \log_2 5$ 13) $\log 10^{35}$ 14) $\log_4 x + \log_4 (x+12) = 3$ - solve for x

$= \log 100$

$= 2$

$= \log_2 8$

$= 3$

$= 35 \cdot \log 10$

$= 35$

$\log_4 (x^2 + 12x) = 3$

(exponentiate base 4)

$x^2 + 12x = 4^3$

$x^2 + 12x - 64 = 0$

$(x+16)(x-4) = 0$

$\cancel{x = -16}$ $x = 4$

extraneous solution.

$4^3 = x^2 + 12x$

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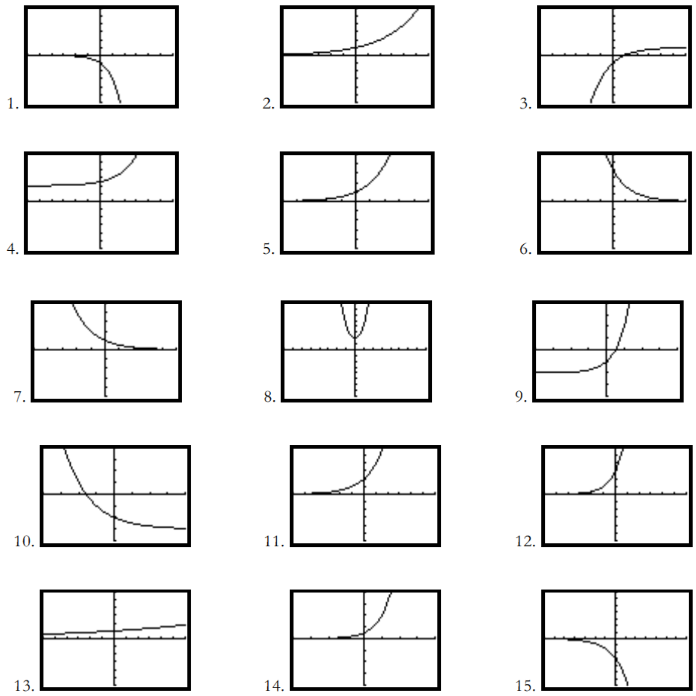
Review of Exponentials and Logarithms - Homework

For each curve below, identify it by the proper equation letter. No calculators.

a. $y = 2^x$	b. $y = 2^{-x}$	c. $y = 2(2^x)$
d. $y = -3^x$	e. $y = -3(2^x)$	f. $y = .5(4^x)$
g. $y = 4^x - 2$	h. $y = .5^x - 3$	i. $y = -2(.5^x) + 1$
j. $y = 3^{x+1}$	k. $y = 2^{x-1} + 2$	l. $y = .5^{x-2}$
m. $y = 2^{x/2}$	n. $y = 1.1^x$	o. $y = 4^x + 4^{-x}$



Solve for x

16. $2^{x-3} = 16$

17. $3^{2x-3} = 81$

18. $5^{3x-3} = 1$

19. $2^{5-2x} = \frac{1}{2}$

20. $10^{5x+6} = \frac{1}{100}$

21. $2^{4x+1} = \sqrt{2}$

22. $27^{3x+3} = 9$

23. $16^{x-3} = 8^{x-3}$

24. $25^{6-2x} = \sqrt{5}$

25. $9^{2x-4} = \left(\frac{1}{27}\right)^{x-3}$

26. $\left(\frac{1}{32}\right)^{x+6} = \left(\frac{1}{8}\right)^{x-2}$

27. $\left(\frac{1}{4}\right)^{2-2x} = \left(\sqrt[3]{2}\right)^{3x+6}$

28. $\log_4 256$

29. $\log_2 8$

30. $\log_8 2$

31. $\log_5 125$

32. $\log_9 27$

33. $\log_7 1$

34. $\log_{25} 5$

35. $\log_8 16$

36. $\log_{\sqrt{3}} 27$

37. $\log_{\frac{1}{5}} \frac{1}{125}$

38. $-5\log_{12} 12$

39. $10^{\log 29}$

40. $\ln e^4$

41. $e^{\ln\sqrt{e}}$

42. $\frac{3\log 10}{2\ln e}$

Solve each equation in terms of x.

43. $\log_3(2x-2)=2$

44. $\log_7(7x-6)=2$

45. $\ln 3x + \ln 3 = 3$

46. $\log_5(x+3) - \log_5 x = 2$

If $a = \log_2 6$ and $b = \log_2 10$, express the following in terms of a and b.

47. $\log_2 24$

48. $\log_2 600$

49. $\log_2 \sqrt[4]{10}$