p 193

the Natural Log Function and Integration - Classwork

The derivative rules which we just learned will now produce the following integration rules:

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + C \text{ and if } u \text{ is a differentiable function of } x,\ \int \frac{1}{u}\,du = \ln|u| + C}$$

① $\int \frac{4}{x}\,dx = 4\ln|x| + C$

② $\frac{1}{5}\int \frac{5}{5x-2}\,dx$

$u = 5x-2$

$du = 5dx$

$= \frac{1}{5}\int \frac{1}{u}\,du$

$= \frac{1}{5}\ln|u| + C$

$= \frac{1}{5}\ln|5x-2| + C$

$\frac{d}{dx}\int_a^x f(t)\,dt$

3) $\frac{1}{-6}\int \frac{4 \cdot -6}{3-6x}\,dx$

$u = 3-6x$

$du = -6\,dx$

$= \frac{4}{-6}\int \frac{1}{u}\,du$

$= -\frac{2}{3}\ln|u| + C$

$= \frac{-2}{3}\ln|3-6x| + C$

4) $\int \frac{7x}{x^2-4}\,dx$

$u = x^2-4$

$du = 2x\,dx$

$= \frac{7}{2}\int \frac{2x\,dx}{x^2-4}$

$= \frac{7}{2}\int \frac{1}{u}\,du$

$= \frac{7}{2}\ln|x^2-4| + C$

$= \frac{7\ln|x^2-4|}{2} + C$

When you take integrals of fractions, you usually think u-substitution with the u being the denominator generating a ln function. But not always.

$-\frac{1}{2}$ 5) $\int \frac{-2x}{\sqrt{16-x^2}}\,dx$ 6) $\int \frac{1}{\sqrt[3]{2x-1}}\,dx$

$\cancel{u = \sqrt{16-x^2}}$

$\cancel{du = \frac{1}{2}(16-x^2)^{-1/2} \cdot -2x\,dx}$

$u = 16-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{2}\int u^{-1/2}\,du = -u^{1/2} + C$

$= -\sqrt{16-x^2} + C$

7) $\int \frac{(\ln x)^4}{x}\,dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx$

$= \int u^4\,du$

$= \frac{u^5}{5} + C$

$= \frac{(\ln x)^5}{5} + C$

8) $\int \frac{x^2-2x+1}{x}\,dx$

$= \int \left(x - 2 + \frac{1}{x}\right) dx$

$= \frac{x^2}{2} - 2x + \ln|x| + C$

9) $\int \tan x \, dx$

$-\int \frac{-\sin x}{\cos x} dx$

$u = \cos x$

$du = -\sin x \, dx$

$= -\int \frac{1}{u} du$

$= -\ln|u| + c$

$= -\ln|\cos x| + C$

10) $\int \cot 3x \, dx$

$= \frac{1}{3} \int 3 \frac{\cos 3x}{\sin 3x} dx$

$u = \sin 3x$

$du = 3\cos 3x$

$= \frac{1}{3} \ln|\sin 3x| + C$

11) $\int \frac{\cos x}{2 + \sin x} dx$

$u = 2 + \sin x$

$= \int \frac{1}{u} du = \ln|2 + \sin x| + C$

12) $\int \frac{1}{\cos^2 x \tan x} dx$

$= \int \frac{\sec^2 x}{\tan x} dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \ln|\tan x| + C$

13) $\displaystyle\int_0^4 \frac{4}{2x+1}\,dx$

14) $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$= \displaystyle\int_{u=1}^{2} \frac{1}{u}\,du$

$= \ln|u|\Big|_1^2$

$= \ln 2 - \ln 1$

$= \ln 2$

15) $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos x}\,dx$

$u = 1+\cos x$

$du = -\sin x\,dx$

$= -\displaystyle\int_{u=2}^{1} \frac{1}{u}\,du$

$= -\left(\ln|u|\right)\Big|_2^1$

$= -\left(\cancel{\ln 1} - \ln 2\right)$

$= \ln 2$