Do now:

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

$$y' = \frac{1}{1+16x^2} \cdot 4$$

$$y'\left(\tfrac{1}{4}\right) = \frac{4}{1+16\left(\frac{1}{16}\right)} = 2$$

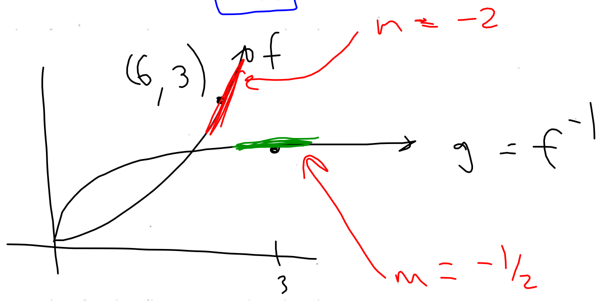
28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $\boxed{g(x) = f^{-1}(x)}$ for all x. What is the value of $\boxed{g'(3)}$?

(A) $-\frac{1}{2}$

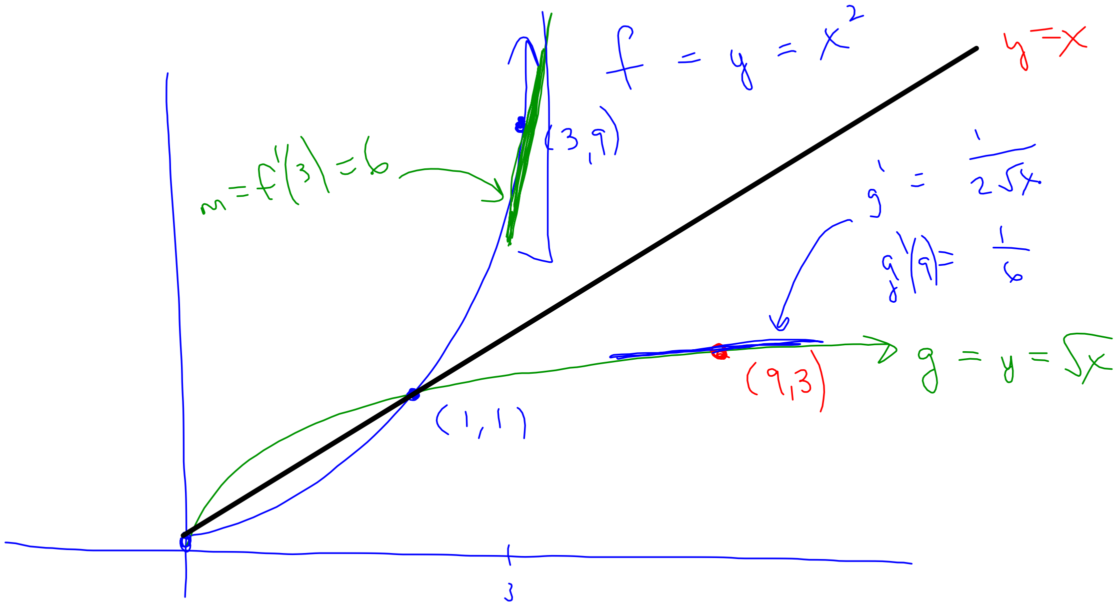
(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$



(E) The value of $g'(3)$ cannot be determined from the information given.



$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$f(3) = 9 \quad f'(3) = 6$

$g = f^{-1}$

what is $g'(9)$?

on g there is a point $(9,3)$

so $g'(9) = 1/6$

28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

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(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

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Derivatives of Inverse Functions - Classwork

General Problem: Find the derivative of the inverse function of $f(x)$ at $x = k$.

Method 1: Simply finding the inverse function. This works when it is easy to generate the inverse function.

a) Find the inverse function by interchanging x and y and solving for y
b) Take the derivative of this new y. That will be the derivative of the inverse function.
c) Plug in your given k value

Method 2: Not finding the inverse function because it is too difficult

a) Find the inverse ~~function~~ relation by interchanging x and y ~~and solving for y~~

b) find $\frac{dy}{dx}$ implicitly

c) Solve for $\frac{dy}{dx}$. It will be in terms of y.

d) Replace the value of k for x in your inverse function from step a above and solve for y

e) Plug that value of y into $\frac{dy}{dx}$

ax

Example: If $f(x) = x^2$, $x \geq 0$, find the derivative of $f^{-1}(x)$ at $x = 4$

<u>**METHOD 1**</u>

a) $y = x^2$, so the inverse is $x = y^2$

therefore $y = \sqrt{x}$ (first quadrant)

b) $y' = \dfrac{1}{2\sqrt{x}}$

c) $y'(4) = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

intersection = ?

Example: If $f(x) = x^2$, $x \geq 0$, find the derivative of $f^{-1}(x)$ at $x = 4$

<u>METHOD 2</u>

a) $y = x^2$, so the inverse is $x = y^2$

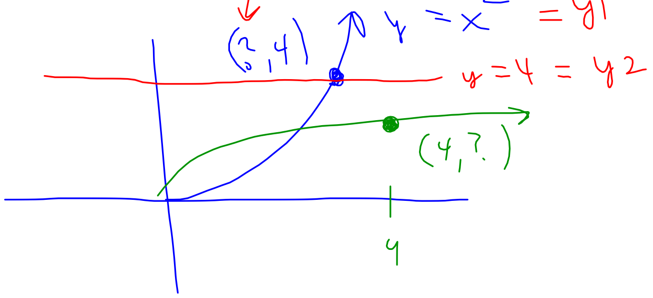
$4 = y^2$

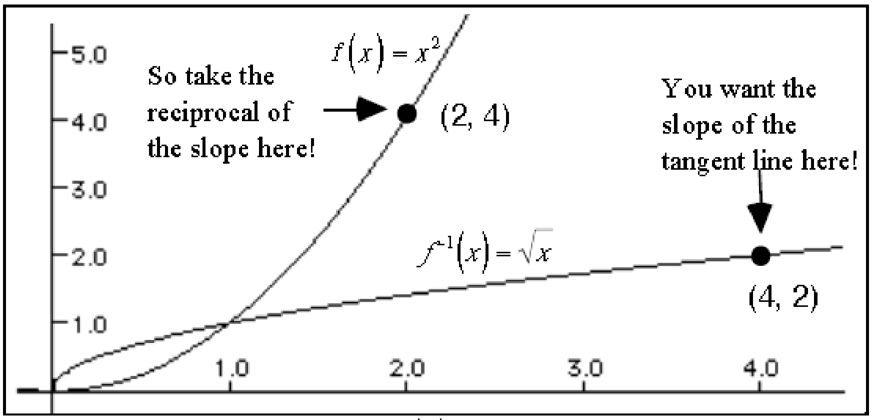
b) $1 = 2y\dfrac{dy}{dx}$

c) $\dfrac{dy}{dx} = \dfrac{1}{2y}$

d) $4 = y^2 \Rightarrow y = 2(\text{quad I})$

e) $\dfrac{dy}{dx} = \dfrac{1}{2y} = \dfrac{1}{2(2)} = \dfrac{1}{4}$





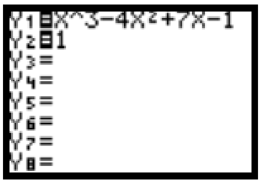
Note: It was necessary to restrict the domain of $f(x)$ to $x \geq 0$ so that its inverse is a function: i.e. that $f(x)$ is one-to-one (passes the horizontal line test).

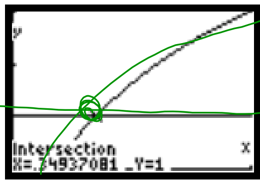
Example: Find the derivative of the inverse function of $f(x)=x^3-4x^2+7x-1$ at $x=1$.

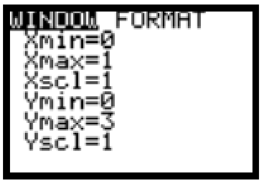
Method 1 will be too difficult. $y=x^3-4x^2+7x-1$ so the inverse is $x=y^3-4y^2+7y-1$

a) $1=\left(3y^2-8y+7\right)\dfrac{dy}{dx}\Rightarrow \boxed{\dfrac{dy}{dx}=\dfrac{1}{3y^2-8y+7}}$

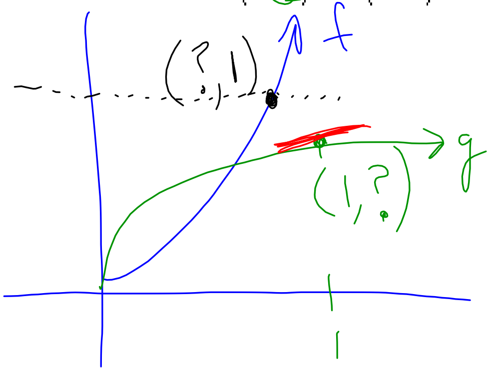
b) Set $y^3-4y^2+7y-1=1$. Graphically, you get $y=.349$.







c) $\dfrac{dy}{dx}=\dfrac{1}{3(.349)^2-8(.349)+7}=.219$



$x = .34937$

$X \to A$

a) $y = x^3 + 1$ @ $x = 9$

inv rel: $x = y^3 + 1$

$8^{2/3} = (\sqrt[3]{8})^2 = \sqrt[3]{(8^2)}$

sol for y: $y = \sqrt[3]{x-1}$

$27^{5/3}$

$y' = \frac{1}{3}(x-1)^{-2/3}$

$y'(9) = \frac{1}{3} \cdot (8)^{-2/3} = \frac{1}{12}$

b) $y = x^3 + 5x - 1$ @ $x = 5$

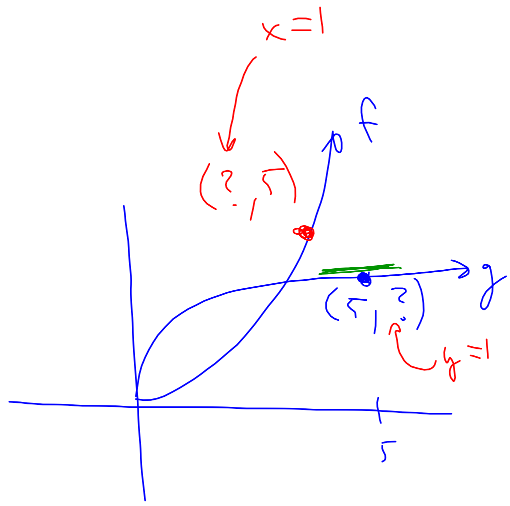
inv rel $\quad x = y^3 + 5y - 1$

impl. diff $\quad 1 = (3y^2 + 5)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{3y^2 + 5}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=5 \\ y=?\ 1}} = \frac{1}{8}$$

$y = 1$



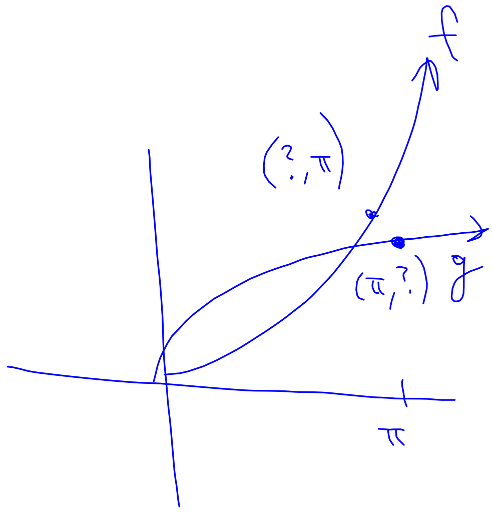
$y = x + \sin x$ @ $x = \pi$

inv rel

$x = y + \sin y$

$1 = (1 + \cos y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{1 + \cos y}$



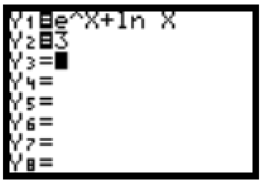
$\left.\frac{dy}{dx}\right|_{\substack{x=\pi \\ y=?}} = \text{Und}$

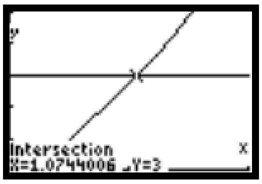
Example: Find the derivative of the inverse function of $f(x) = e^x + \ln x$ at $x = 3$

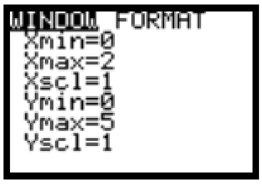
Method 1 will be too difficult. $y = e^x + \ln x$ so the inverse is $x = e^y + \ln y$

a) $1 = \left(e^y + \frac{1}{y}\right)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{e^y + \frac{1}{y}}$ or $\frac{dy}{dx} = \frac{y}{ye^y + 1}$

b) Set $e^y + \ln y = 3$. Graphically, you get $y = 1.0744$.







c) $\frac{dy}{dx} = \frac{1}{e^{1.0744} + \frac{1}{1.0744}} = .259$

Note: After you graphically intersect, you can easily get the answer by $\frac{1}{\text{nDeriv(Y1,X,X)}}$

Example: Find the derivative of the inverse function of $y = e^{x^2}$, $x > 0$

Inverse Function: $x = e^{y^2}$

$\ln x = y^2$

Solution: $y = \sqrt{\ln x} = (\ln x)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(\ln x)^{-1/2}\frac{1}{x} = \frac{1}{2x(\ln x)^{1/2}}$$

Sample Problems: Find the derivative of the inverse function of (use method 2 only if method 1 won't work)

a) $y = x^3 + 1$ at $x = 9$. b) $y = x^3 + 5x - 1$ at $x = 5$ c) $y = x + \sin x$ at $x = \pi$.

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a) $y = x^3 + 1$ at $x = 9$. b) $y = x^3 + 5x - 1$ at $x = 5$ c) $y = x + \sin x$ at $x = \pi$.

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