Solve the differential eqn:

$$f''(x) = 2, \quad f'(4) = 1, \quad f(-1) = 2$$

$$f'(x) = 2x + C = 2x - 7$$

$$f'(4) = 2(4) + C = 1$$

$$C = -7$$

$$f(x) = x^2 - 7x + C = x^2 - 7x - 6$$

$$f(-1) = 1 + 7 + C = 2$$

$$C = -6$$

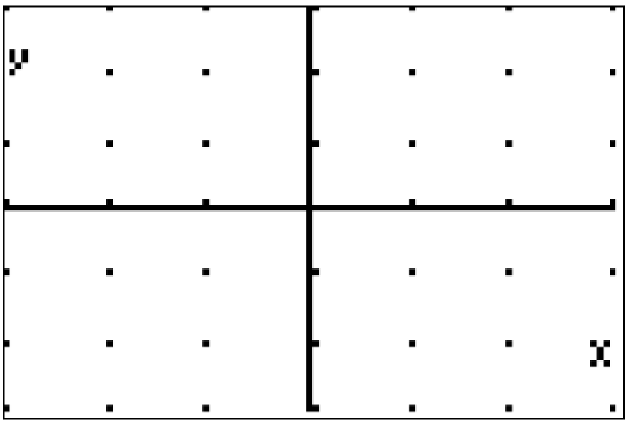
Slope Fields, DEQ's, Exponential Growth PRETEST **Name** ______________________________

1. Complete the chart and create a slope field for the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$

y

x

	−2	−1	0	1	2
2	-2	−4	und	4	2
1					
0					
−1					
−2					



Solve the following differential equations:

2. $\frac{dy}{dx} = \frac{x-1}{2y}$

$\int 2y \, dy = \int (x-1) \, dx$

$y^2 = \frac{x^2}{2} - x + C$

$y = \pm\sqrt{\frac{x^2}{2} - x + C}$

3. $\frac{dy}{dx} = y\cos x$

$\frac{dy}{y} = \cos x \, dx$

4. A rash is expanding in area at the rate that is proportional to its current area. If its size at 2 PM is 2 square inches and at 2.30 PM, its size is 2.75 square inches, what will be its size at 5 PM? Show all work.

Differential Equations by Separation of Variables - Classwork

A differential equation ~~will be~~ in the form of $\frac{dy}{dx} = f(x) \cdot g(y)$. ~~In order to solve it, you~~ must put it in the form of $g(y) \cdot dy = f(x) \cdot dx$ allowing you to integrate. Your goal is to get an equation in the form of $y = h(x)$

1) $\frac{dy}{dx} = \frac{2x}{y}$

$y\,dy = 2x\,dx$

$\frac{y^2}{2} = x^2 + C$

$y^2 = 2x^2 + C$

$y = \pm\sqrt{2x^2 + C}$

2) $\frac{dy}{dx} = \frac{y^2}{1}$

$\frac{dy}{y^2} = dx$

$-y^{-1} = x + C$

$y^{-1} = -x + C$

$\frac{1}{y} = -x + C$

$y = \frac{1}{-x + C}$

3) $\frac{dy}{dx} = \frac{x + \sin(x)}{3y^2}$

$3y^2\,dy = (x + \sin x)\,dx$

$y^3 = \frac{x^2}{2} - \cos x + C$

$y = \sqrt[3]{\frac{x^2}{2} - \cos x + C}$

4) $\frac{dy}{dx} = 4y$

$$\frac{dy}{y} = 4\,dx$$

$$\ln|y| = 4x + C$$

$$e^{\ln y} = e^{4x+C}$$

$$y = e^{4x} e^{C}$$

$$\boxed{y = Ce^{4x}}$$

5. $\frac{dy}{dx} = ky$

$$\frac{dy}{y} = k\,dx$$

$$\ln|y| = kx + C$$

$$y = Ce^{kx}$$

6) $\frac{dy}{dx} = xy$

$$\frac{dy}{y} = x\,dx$$

$$\ln y = \frac{x^2}{2} + C$$

$$y = e^{\frac{x^2}{2}+C}$$

$$\boxed{y = Ce^{x^2/2}}$$

7. $\frac{du}{dt} = e^{u+2t}$

8. $\frac{dx}{dt} = 1 + t - x - tx$

$$\frac{du}{dt} = e^{u} e^{2t}$$

$$\frac{du}{e^{u}} = e^{2t}\,dt$$

$$e^{-u}\,du$$

$$-e^{-u} = \frac{1}{2}e^{2t} + C$$

$$\frac{1}{e^{u}} = -\frac{1}{2}e^{2t} + C$$

$$e^{u} = \frac{1}{-\frac{1}{2}e^{2t} + C}$$

$$u = \ln\left(\frac{1}{-\frac{1}{2}e^{2t} + C}\right) = -\ln\left(C - \frac{1}{2}e^{2t}\right)$$

8. $\frac{dx}{dt} = 1 + t - x - tx$

$$\frac{dx}{dt} = 1(1+t) - x(1+t)$$

$$\frac{dx}{dt} = (1-x)(1+t)$$

$$\frac{dx}{1-x} = (1+t)\,dt$$

$u = 1-x$
$du = -1dx$

$$-\int \frac{1}{u}\,du = t + \frac{t^2}{2} + C$$

$$-\ln|1-x| = t + \frac{t^2}{2} + C$$

$$\ln|1-x| = -t - \frac{t^2}{2} + c$$

$$1-x = e^{-t-\frac{t^2}{2}+c}$$

$$1-x = Ce^{-t-\frac{t^2}{2}}$$

$$x = 1 - Ce^{-t-\frac{t^2}{2}}$$

Example of factoring by grouping -- this technique is used in example #8

$$\underbrace{2a^3 - a^2 b}\ +\ \underbrace{10a - 5b}$$

$$a^2(2a - b) + 5(2a - b)$$

$$(a^2 + 5)(2a - b)$$

p213 #1, 3, 5

Questions?

$$x \frac{dy}{dx} = y$$

(3) $$\frac{dy}{y} = \frac{1}{x}\,dx$$

$$\ln|y| = \ln|x| + C$$

$$y = e^{\ln|x| + C}$$

$$y = e^{\ln x} e^{c}$$

$$y = Cx$$

5 $$yy' = \sin x$$

$$y\frac{dy}{dx} = \sin x$$

$$y\,dy = \sin x\,dx$$

$$\frac{y^2}{2} = -\cos x + C$$

$$y^2 = -2\cos x + C$$

$$y = \pm\sqrt{C - 2\cos x}$$

$x \cdot \frac{1}{e^t} \frac{dx}{dt} = 1$

Find the solution of the differential equation that satisfies the given condition.

9. $\boxed{xe^{-t}\frac{dx}{dt} = 1}$, $\boxed{x(0) = 1}$ → find C

10. $\frac{dy}{dx} = \frac{1+x}{xy}$, $y(1) = -4$

$x\,dx = e^t\,dt$

$\frac{x^2}{2} = e^t + C$

$x^2 = 2e^t + C$

$x(t) = \pm\sqrt{2e^t + C}$ → $x = \pm\sqrt{2e^t - 1}$

$X = \sqrt{2e^t - 1}$

$x(0) = \pm\sqrt{2e^0 + C} = 1$

$\sqrt{2 + C} = 1$

$2 + C = 1$

$C = -1$

(10) $\frac{dy}{dx} = \frac{1+x}{xy}$; $y(1) = -4$

$$y\,dy = \frac{1+x}{x}\,dx$$

$$y\,dy = \left(\frac{1}{x}+1\right)dx$$

$$\frac{y^2}{2} = \ln|x| + x + C$$

$$y^2 = 2\ln|x| + 2x + C$$

$$y = \pm\sqrt{2\ln|x| + 2x + C}$$

$$y(1) = \pm\sqrt{2\ln 1 + 2 + C} = -4$$

$$\sqrt{2+c} = -4$$

$$2+c = 16$$

$$c = 14$$

$$y = -\sqrt{2\ln|x| + 2x + 14}$$

(11) $\frac{dy}{dx} = y^2 + 1 \; ; \; y(1) = 0$

p207 $\frac{dy}{1+y^2} = 1\,dx$

$a=1$ $\quad u = y$

$\tan^{-1} y = x + c$

$y = \tan(x + c)$

$y(1) = \tan(1 + c) = 0$

$1 + c = 0$

$c = -1$

$$\boxed{y = \tan(x - 1)}$$

(12) $x + 2y\sqrt{x^2+1}\,\frac{dy}{dx} = 0;\ y(0)=$

$$2y\sqrt{x^2+1}\,\frac{dy}{dx} = -x$$

$$2y\,dy = \frac{-x}{\sqrt{x^2+1}}\,dx$$

$u = x^2+1$
$du = 2x\,dx$

$$y^2 = -\frac{1}{2}\int u^{-1/2}\,du$$

$$y^2 = -u^{1/2} + C$$

$$y = \sqrt{C - \sqrt{x^2+1}}$$

$$y(0) = \sqrt{C - \sqrt{1}} = 1$$

$$\sqrt{C-1} = 1$$

$$C = 2$$

$$y = \sqrt{2 - \sqrt{x^2+1}}$$