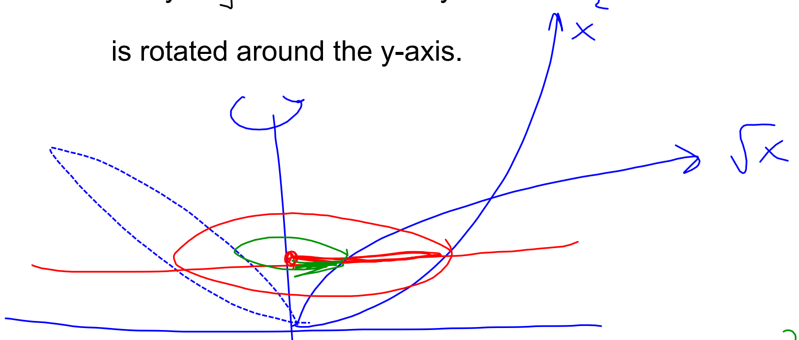
WARMUP (review)

Compute the volume of the solid when the region enclosed by

$x = y^2$ $\quad y = \sqrt{x} \quad$ and $\quad y = x^2 \quad$ $x = \sqrt{y}$

is rotated around the y-axis.

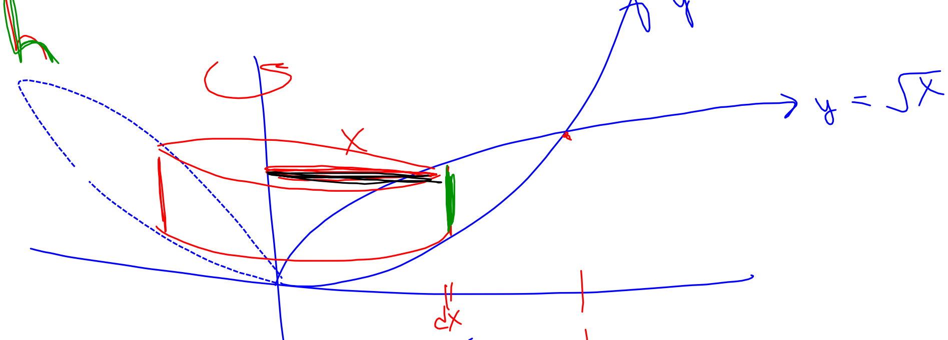


$R = \sqrt{y} \qquad r = y^2$

$$V = \pi \int_{y=0}^{1} \left(y - y^4\right) dy$$

Surface Area of Walls

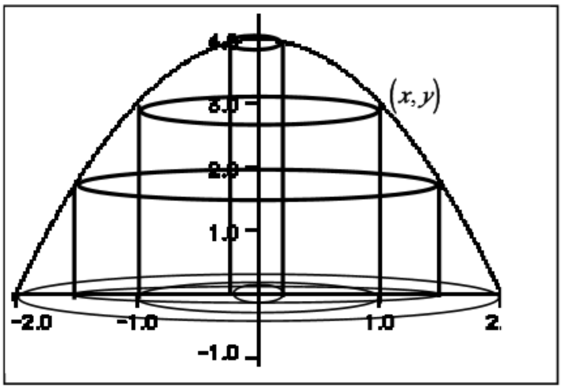
$2\pi r \cdot h$



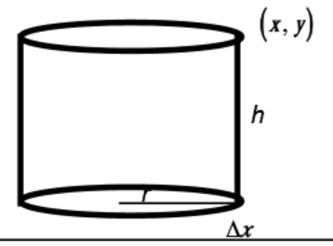
$$V = 2\pi \int_{x=0}^{1} x \cdot \left(\sqrt{x} - x^2\right) dx$$

http://web.monroecc.edu/manila/webfiles/calcNSF/JavaCode/other/myShell1.htm

Volume by Cylindrical Shells - Classwork



There is another way to determine the volume of a curve that is rotated about an axis - the method of cylindrical shells. Let us take some function $y = f(x)$ and rotate it about the y-axis as shown above. Instead of creating a disk, we will create a cylinder. This cylinder is very thin - its sides are like paper. We call it a cylindrical shell. We can draw many such cylinders (above three are drawn). Note that they all have different heights and different radii.



The surface area of this shell is
$2\pi(\text{radius})(\text{height})$
Since this shell has thickness Δx
The volume of the shell can be expressed
$V = 2\pi(\text{radius})(\text{height})\Delta x$

Since we are using infinitely thin shells $\left(\lim\limits_{\Delta x \to 0}\right)$, we can say that the volume of our rotated region is

$$V = \lim_{n\to\infty}\left[\sum_{i=1}^{n} 2\pi \cdot (\text{radius}_i)(\text{height}_i)\Delta x\right]$$

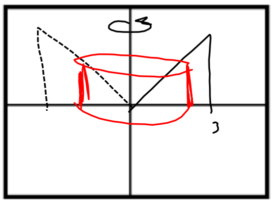
$\displaystyle\int_{x=a}^{x=b} 2\pi r \cdot h\, dx$ if rotated about y-axis or vertical line

$\displaystyle\int_{y=a}^{y=b} 2\pi r \cdot h\, dy$ if rotated about x-axis or horizontal line

So when you attempt a problem by cylindrical shells, your job is to determine both the radius and height. Note that in using the disk/washer method, it is usually better to rotate about the x-axis or lines parallel to the x-axis (y = constant) because the outside and inside radii are expressed as a vertical distance which will be a function of x. The shell method's advantage is usually in rotating about the y-axis or lines parallel to the y-axis (x = constant) because the radius is a horizontal distance given some function of x and the height is the vertical distance which is a function of x. So from now on, it is recommended that you use shells when rotating around the y-axis or lines parallel to the y-axis.

1. Find the volume if the region enclosing $y = 2x, y = 0,\ x = 3$ is rotated about the

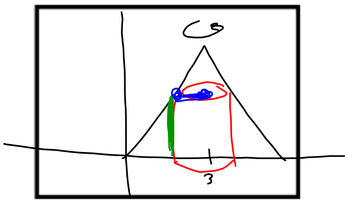
a) the y-axis



$r =$ x $h =$ $2x$

$V =$ $2\pi \int_{x=0}^{3} x \cdot 2x \, dx$

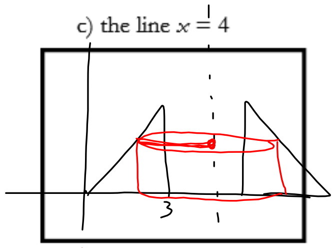
b) the line $x = 3$



$r =$ $3 - x$ $h =$ $2x$

$V =$ $2\pi \int_{x=0}^{3} (3-x)(2x)\, dx$

c) the line $x = 4$



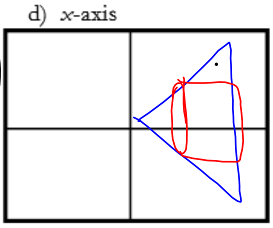
$r =$ $4 - x$ $h =$ $2x$

$V =$ $2\pi \int_{x=0}^{3} (4-x)(2x)\, dx$

$y = 2x$

$\frac{1}{2}y = x$

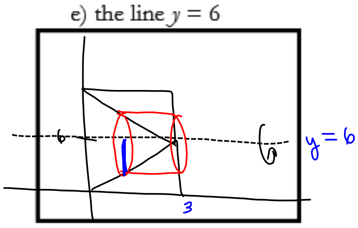
d) x-axis



R = y $h =$ $3 - \frac{1}{2}y$

$V =$ $2\pi \int_{y=0}^{6} y\left(3 - \frac{1}{2}y\right) dy$

e) the line $y = 6$

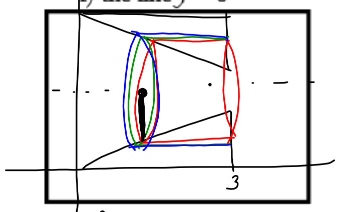


$y = 6$

R = $6 - y$ $h =$ $3 - \frac{1}{2}y$

$V =$ $2\pi \int_{y=0}^{6} (6-y)\left(3 - \frac{1}{2}y\right) dy$

f) the line $y = 8$



$y = 8$

R = $8 - y$ $h =$ $3 - \frac{1}{2}y$

$V =$ $2\pi \int_{y=0}^{6} (8-y)\left(3 - \frac{1}{2}y\right) dy$

Again, the shell method's greatest advantage is rotating around the y-axis when the problem can be in terms of x. A downside of the shell method is that you may not be able to integrate using the Fundamental Theorem. Calculators are of a real advantage here.

2. Find the volume if the region enclosing $y = x^2, y = 0, x = 4$ is rotated about the

a) y-axis

$r =$ ________ $h =$ ________

$V =$ ____________________

b) the line $x = 4$

$r =$ ________ $h =$ ________

$V =$ ____________________

3. Find the volume if the region enclosing $y = x^3, y = 8, x = 0$ is rotated about the

a) y-axis

$r =$ ________ $h =$ ________

$V =$ ____________________

Volume by Cylindrical Shells - Homework

1. Find the volume if the region enclosing $y = -x^2 + 4x + 3, x = 0, x = 4, y = 0$ is rotated about

a) y-axis

$r =$ ________ $h =$ ________

$V =$ ____________________

b) the line $x = 4$

$r =$ ________ $h =$ ________

$V =$ ____________________

c) the line $x = 5$

$r =$ ________ $h =$ ________

$V =$ ____________________

2. Find the volume if the rfirst quadrant region enclosing $y = x^2, y = 8 - x^2$ is rotated about

a) y-axis

$r =$ ________ $h =$ ________

$V =$ ____________________

b) the line $x = 2$

$r =$ ________ $h =$ ________

$V =$ ____________________

c) the line $x = -1$

$r =$ ________ $h =$ ________

$V =$ ____________________

3. Find the volume if the region enclosed by $y = \sin x, y = 0$ on $[0, \pi]$ is rotated about

a) y-axis

$r =$ ________ $h =$ ________

$V =$ ____________________

b) the line $x = \pi$

$r =$ ________ $h =$ ________

$V =$ ____________________

c) the line $x = 2\pi$

$r =$ ________ $h =$ ________

$V =$ ____________________