Inverse Trig Functions - Homework

1) Evaluate each of the following:

a. $\boxed{\arccos\frac{1}{2} \quad \frac{\pi}{3}}$

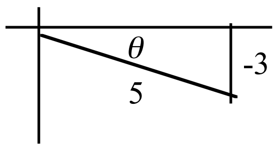
b. $\boxed{\cot^{-1}\sqrt{3} \quad \frac{\pi}{6}}$

c. $\boxed{\sin^{-1}\frac{-\sqrt{3}}{2} \quad -\frac{\pi}{3}}$

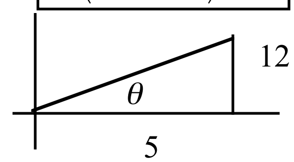
d. $\boxed{\sec^{-1}-\frac{2\sqrt{3}}{3} \quad \frac{5\pi}{6}}$

2) Evaluate the following. Make a picture to describe the situation.

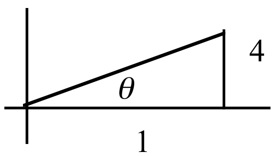
a. $\boxed{\cos\left(\arcsin\frac{-3}{5}\right) = \frac{4}{5}}$



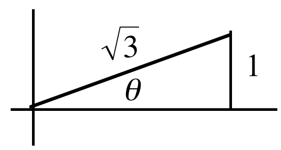
b. $\boxed{\sin\left(\arctan\frac{12}{5}\right) = \frac{12}{13}}$



c. $\boxed{\csc\left(\cot^{-1}4\right) = \sqrt{17}}$

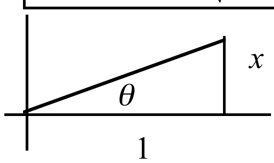


d. $\boxed{\tan\left(\csc^{-1}\sqrt{3}\right) = \frac{\sqrt{2}}{2}}$

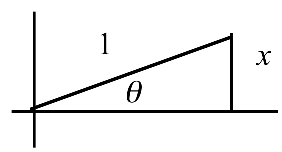


3) Evaluate the following. Make a picture to describe the situation.

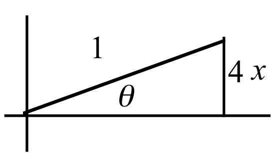
a. $\boxed{\cos\left(\tan^{-1}x\right) = \frac{1}{\sqrt{1+x^2}}}$



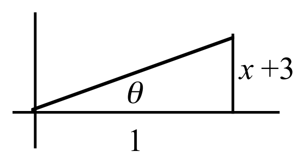
b. $\boxed{\sec\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}}$



c. $\boxed{\tan\left(\sin^{-1}4x\right) = \frac{4x}{\sqrt{1-16x^2}}}$



d. $\boxed{\cos\left(\tan^{-1}(x+3)\right) = \frac{1}{\sqrt{x^2+6x+10}}}$



4) Find the derivatives of

a. $\boxed{y = \cos^{-1}(3x) \quad \frac{dy}{dx} = \frac{-3}{\sqrt{1-9x^2}}}$

b. $\boxed{y = \sin^{-1}\left(x^2-1\right) \quad \frac{dy}{dx} = \frac{2x}{\sqrt{1-\left(x^2-1\right)^2}} = \frac{2x}{\sqrt{2x^2-x^4}}}$

c. $y = \left(\tan^{-1} 2x\right)^5$

$\dfrac{dy}{dx} = 5\left(\tan^{-1} 2x\right)^4 \dfrac{2}{1+4x^2} = \dfrac{10\left(\tan^{-1} 2x\right)^4}{1+4x^2}$

d. $y = \sqrt{\left(\cos^{-1} 10x\right)}$

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{\left(\cos^{-1} 10x\right)}} \dfrac{-10}{\sqrt{1-100x^2}}$

$\dfrac{dy}{dx} = \dfrac{-5}{\sqrt{\left(\cos^{-1} 10x\right)}\sqrt{1-100x^2}}$

e. $y = \arctan\sqrt{x}$

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}\left(1+x\right)}$

f. $y = \sin\left(\cos^{-1} t\right)$

$\dfrac{dy}{dt} = \cos\left(\cos^{-1} t\right)\dfrac{-1}{\sqrt{1-t^2}}$

$\dfrac{dy}{dt} = \dfrac{-t}{\sqrt{1-t^2}}$

5) Find any relative extrema of $y = \arcsin x - x$

$y = \arcsin x - x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}} - 1 = 0$

$x = 0$, Critical points $x = -1, 1 \Rightarrow \dfrac{dy}{dx} > 0 \ \left(-1,0\right)\cup\left(0,1\right)$ No relative extrema

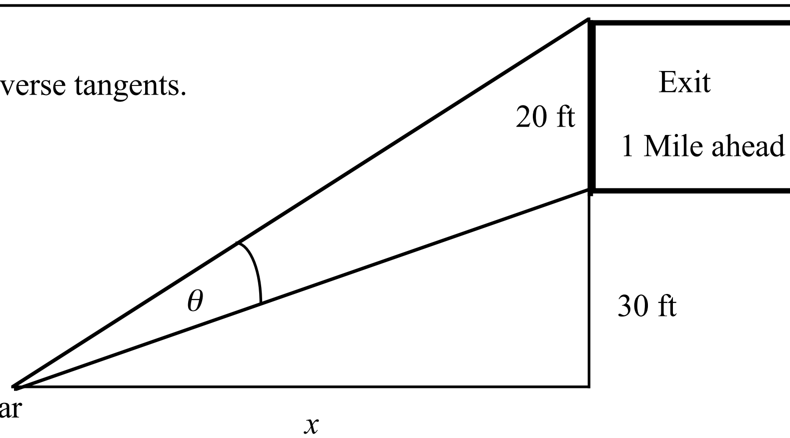
6) The base of a 20 foot tall exit sign is 30 feet above the driver's eye level. When cars are far away, the sign is hard to read because of the distance. When they are close, the sign is hard to read because the driver has to look up at a steep angle. The sign is easiest to read when the distance x is such that the angle θ at the driver's eye is as large as possible.

a) Write θ as the difference of 2 inverse tangents.

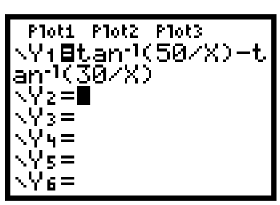
$\theta = \tan^{-1}\dfrac{50}{x} - \tan^{-1}\dfrac{30}{x}$

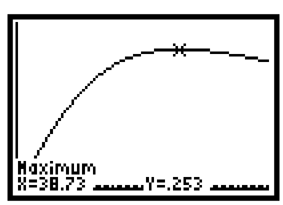
b) Write an equation for $\dfrac{d\theta}{dx}$

$\dfrac{d\theta}{dx} = \dfrac{-50}{x^2+2500} + \dfrac{30}{x^2+900}$



c) The sign is easiest to read at the value of x where θ stops increasing and starts decreasing. This happens when $\dfrac{d\theta}{dx} = 0$. Find x and confirm using the calculator. $5x^2 + 4500 = 3x^2 + 7500 \Rightarrow x = 38.730$ ft.





Inverse Trig Functions Integration - Homework

1. $\displaystyle\int \frac{dx}{1+4x^2}$

$\boxed{\begin{array}{l} a=1, u=2x, du=2dx \\ \dfrac{1}{2}\tan^{-1}2x+C \end{array}}$

2. $\displaystyle\int \frac{dx}{\sqrt{4-x^2}}$

$\boxed{\begin{array}{l} a=2, u=x, du=dx \\ \sin^{-1}\dfrac{x}{2}+C \end{array}}$

3. $\displaystyle\int \frac{dx}{4+(x-1)^2}$

$\boxed{\begin{array}{l} a=2, u=x-1, du=dx \\ \dfrac{1}{2}\tan^{-1}\dfrac{(x-1)}{2}+C \end{array}}$

4. $\displaystyle\int \frac{t}{\sqrt{1-t^4}}\,dt$

$\boxed{\begin{array}{l} a=1, u=t^2, du=2tdt \\ \dfrac{1}{2}\sin^{-1}t^2+C \end{array}}$

5. $\displaystyle\int \frac{x}{x^4+16}\,dx$

$\boxed{\begin{array}{l} a=4, u=x^2, du=2xdx \\ \dfrac{1}{8}\tan^{-1}\dfrac{x^2}{4}+C \end{array}}$

6. $\displaystyle\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$\boxed{\begin{array}{l} u=\sin^{-1}x \quad du=\dfrac{1}{\sqrt{1-x^2}}\,dx \\ \dfrac{\left(\sin^{-1}x\right)^2}{2}+C \end{array}}$

7. $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

$\boxed{\begin{array}{l} a=1, u=\cos x, du=-\sin xdx \\ -\tan^{-1}(\cos x)+C \end{array}}$

8. $\displaystyle\int \frac{e^{2x}}{9+e^{4x}}\,dx$

$\boxed{\begin{array}{l} a=3, u=e^{2x}, du=2e^{2x}dx \\ \dfrac{1}{6}\tan^{-1}\left(\dfrac{e^{2x}}{3}\right)+C \end{array}}$

9. $\displaystyle\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx$

$\boxed{\begin{array}{l} a=1, u=2x, du=2dx \\ \dfrac{1}{2}\tan^{-1}(2x)\Big|_0^{\sqrt{3}/2}=\tan^{-1}\sqrt{3}=\dfrac{\pi}{6} \end{array}}$

10. Find the area of the region bounded by the curves $y=\dfrac{1}{\sqrt{4-x^2}}$, $y=0$, $x=0$, $x=1$

$\boxed{\sin^{-1}\dfrac{x}{2}\Big|_0^1=\dfrac{\pi}{6}}$