

A.P. Calculus - ln and e Practice test

1) $y = \ln(-14x^3)$

2) $y = (\ln 2x)^5$

3) $y = \ln(4 - x\sqrt{2})$

4) $y = \ln(\ln(\cos x))$

5) $y = \frac{x^4}{1 - 2\ln x}$

6) $y = \ln^4 \sqrt{\frac{x^2 + 1}{x^2 - 1}}$

7) $y = e^{-4x+4}$

8) $y = 2e^{2x/3}$

9) $y = xe^2 - e^x$

10) $y = 4^{\sqrt{2x-1}}$

11) $y = \pi^x x^\pi$

12) $x^4 + e^{xy} - y^2 = 20$

Find the integral or definite integral as indicated

13) $\int \frac{1}{5-x} dx$

14) $\int \frac{7x dx}{x^2 - 8}$

15) $\int \frac{8x^3 - 7x^2 - 1}{2x} dx$

16) $\int \frac{x^2 - 5x + 2}{x - 2} dx$

17) $\int \frac{(\ln x)^3}{2x} dx$

18) $\int (e^{4x} + e) dx$

19) $\int 2e^x(1 + e^x)^4 dx$

20) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

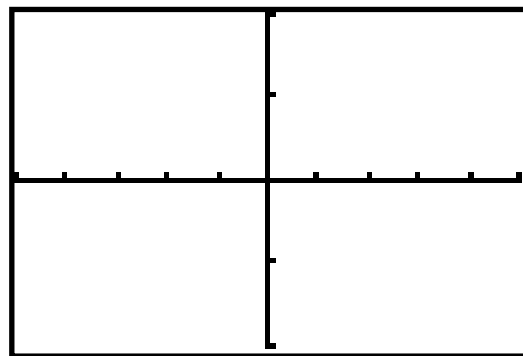
21) $\int (e^x - e^{-x})^2 dx$

22) $\int \pi e^{\cos x} \sin x dx$

$$23) \int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

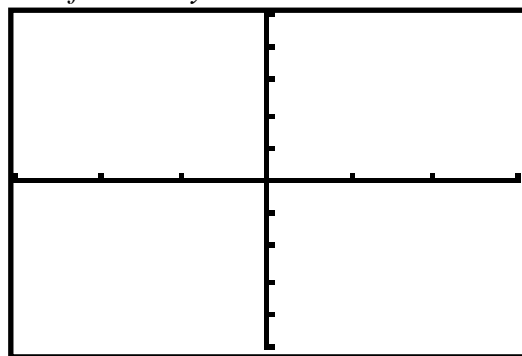
$$24) \int_1^{e^2} \frac{2}{x} dx$$

- 25) Find relative extrema and inflection points of $y = xe^{-x}$ Use calculus methods and be sure that your work is understandable.

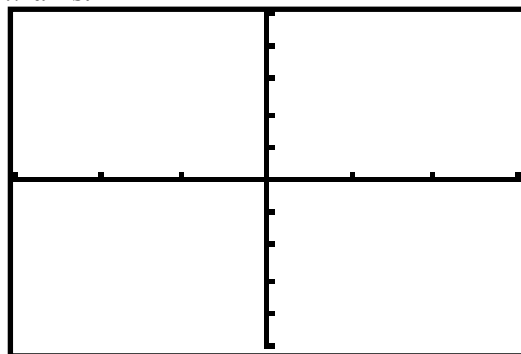


26) Given $f(x) = e^{x/2}$

- a) Find the **area** of the region R bounded by the line $y = e$, the graph of f and the y -axis.



- b) Find the **volume** of the solid generated by revolving R about the x -axis.



A.P. Calculus - ln and e Practice test - Solutions

1) $y = \ln(-14x^3)$

$$\frac{dy}{dx} = \frac{1}{-14x^3}(-42x^2) = \frac{-3}{x}$$

3) $y = \ln(4 - x\sqrt{2})$

$$\frac{dy}{dx} = \frac{1}{4 - x\sqrt{2}}(-\sqrt{2}) = \frac{-\sqrt{2}}{4 - x\sqrt{2}}$$

5) $y = \frac{x^4}{1 - 2\ln x}$

$$\frac{dy}{dx} = \frac{(1 - 2\ln x)(4x^3) - x^4\left(\frac{-2}{x}\right)}{(1 - 2\ln x)^2} = \frac{6x^3 - 8x^3 \ln x}{(1 - 2\ln x)^2}$$

7) $y = e^{-4x+4}$

$$\frac{dy}{dx} = -4e^{-4x+4}$$

9) $y = xe^x - e^x$

$$\frac{dy}{dx} = e^x - e^x$$

11) $y = \pi^x \cdot x^\pi$

$$\frac{dy}{dx} = \pi^{x+1}x^{\pi-1} + x^\pi \pi^x \ln \pi$$

23) $\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$

$$u = \sqrt{x}, du = \frac{1}{2\sqrt{x}} \quad x=1, u=1 \quad x=4, u=2$$

$$e^u \Big|_1^2 = e^2 - e$$

25) Find relative extrema and inflection points of $y = xe^{-x}$. Use calculus methods and be sure that your work is understandable.

$$y' = -xe^{-x} + e^{-x} = 0$$

$$e^{-x}(1-x) = 0$$

$$x=1 \quad \text{Rel. Max at } \left(1, \frac{1}{e}\right)$$

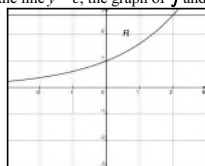
$$y'' = xe^{-x} - e^{-x} - e^{-x}$$

$$e^{-x}(x-2) = 0$$

$$x=2 \quad \text{Inf. Pt. at } \left(2, \frac{1}{e^2}\right)$$

26) Given $f(x) = e^x$

a) Find the **area** of the region R bounded by the line $y = e$, the graph of f and the y-axis.



$$A = \int_0^1 (e - e^{t^2}) dx$$

$$A = ex - 2e^{t^2} \Big|_0^1 = 2e - 2e - (0 - 2) = 2$$

b) Find the **volume** of the solid generated by revolving R about the x-axis.

$$V = \pi \int_0^1 (e^2 - e^t) dx$$

$$V = \pi (e^2 x - e^t) \Big|_0^1 = (e^2 + 1)\pi = 8.389\pi = 26.355$$

Find the integral or definite integral as indicated

13) $\int \frac{1}{5-x} dx$

$$-\ln|5-x| + C$$

15) $\int \frac{8x^3 - 7x^2 - 1}{2x} dx$

$$\int \left(4x^2 - \frac{7x}{2} - \frac{1}{2x}\right) dx$$

$$\frac{4x^3}{3} - \frac{7x^2}{4} - \frac{1}{2} \ln|x| + C$$

17) $\int \frac{(\ln x)^3}{2x} dx$

$$u = \ln x \quad du = \frac{1}{x} dx$$

$$\frac{1}{2} \frac{(\ln x)^4}{4} = \frac{(\ln x)^4}{8} + C$$

19) $\int 2e^x(1+e^x)^4 dx$

$$u = 1 + e^x \quad du = e^x dx$$

$$2 \int u^4 du$$

$$\frac{2}{5} (1 + e^x)^5 + C$$

21) $\int (e^x - e^{-x})^2 dx$

$$\int (e^{2x} - 2 + e^{-2x}) dx$$

$$\frac{e^{2x}}{2} - 2x - \frac{e^{-2x}}{2} + C$$

14) $\int \frac{7x}{x^2-8} dx$

$$\frac{7}{2} \ln|x^2-8| + C$$

16) $\int \frac{x^2-5x+2}{x-2} dx$

$$\int \left(x - 3 - \frac{4}{x-2}\right) dx \text{ (long division)}$$

$$\frac{x^2}{2} - 3x - 4 \ln|x-2| + C$$

18) $\int (e^{4x} + e) dx$

$$\frac{1}{4} e^{4x} + ex + C$$

20) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

$$u = e^x - e^{-x} \quad du = (e^x + e^{-x}) dx$$

$$\int \frac{du}{u}$$

$$\ln|e^x - e^{-x}| + C$$

22) $\int \pi e^{\cos x} \sin x dx$

$$u = \cos x, du = -\sin x dx$$

$$-\pi \int e^u du$$

$$-\pi e^{\cos x} + C$$